Proper Time Oscillator

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Time and Space Symmetry

- A simple harmonic oscillator has oscillation in space but not in time.
- Following the spirit of relativity, can matter has oscillation in time?
- Assuming a particle can oscillate in proper time:

  1. Spacetime around is the Schwarzschild field.
  2. Reconcile same properties of a quantum field (bosons and fermions).
  3. Proper time oscillation satisfies an uncertainty relation analogous to position-momentum uncertainty relation.
  4. Higgs boson as a proper time oscillator.

Fluctuation of Neutrino's Arrival Time

- Study in quantum gravity (lightcone fluctuation) evaluates the accumulated uncertainty effects of a neutrino's travel time and distance in fluctuating spacetime.

- Suggested uncertainty follows a power-law depending on the neutrino's energy, i.e., $\Delta t' \propto l^m E^n$. , where m and n are factors to be established by experiments or theoretical predictions.

- **Uncertainty derived from temporal oscillation is $\Delta t' \propto E^{1/2}$ akin to the power-law from lightcone fluctuation.**

References

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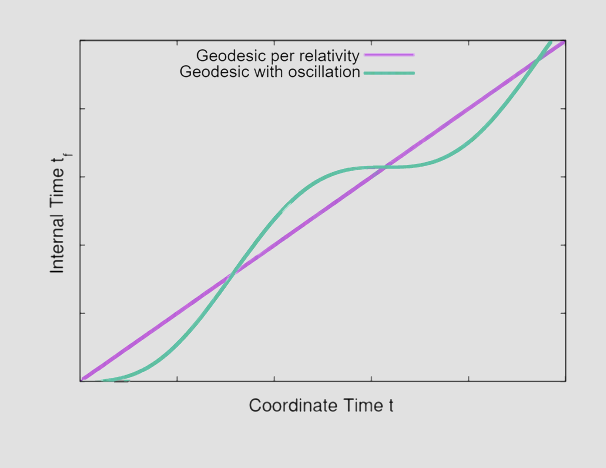
Oscillator in Space

Analogy as a particle traveling at average $\mathbf{v}$, but oscillate with angular frequency ω and amplitude $\mathring{\mathbf{X}}$,

$$\mathring{\mathbf{x}}_f = \mathbf{v}t - \mathring{\mathbf{X}} \sin(\omega t). \tag{1}$$

- Replace motions in space with motions in time. **Assume proper time of a stationary particle also oscillates**

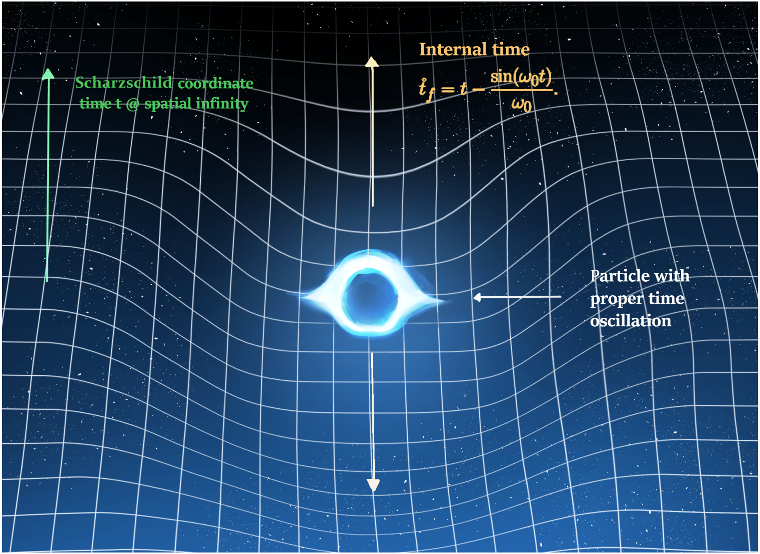
$$\mathring{t}_f = t - \mathring{T}_0 \sin(\omega_0 t), \quad \mathring{T}_0 = 1/\omega_0. \tag{2}$$



- Particle appear to travel along timelike geodesic if the instrument used not sensitive enough.
- **Particles never travel backward in time**.
- **internal time evolves tightly with the coordinate time.**.

Schwarzschild Field

Spacetime outside proper time oscillator is Schwarzschild


Internal time
$t_f = t - \frac{\sin(\omega_0 t)}{\omega_0}$.
Scharzschild coordinate time t @ spatial infinity
Particle with proper time oscillation

Past Examples Schwarzschild Metric without General Relativity

- Lenz W 1944 unpublished work cited by Sommerfeld A
- Schiff L I 1960 On experimental tests of the general theory of relativity Am. J. Phys. 28 340-343
- Hamilton A J S and Lisle J P 2004 The river model of black holes Am. J. Phys
- Laschkarew W 1926 Zur Theorie der Gravitation Z.Physik 35 473-476
- Czerniawski J The possibility of a simple derivation of the Schwarzschild metric rXiv:gr-qc/0611104

$$ds^2 = [1 - \underline{v}^2]dt^2 - [1 - \underline{v}^2]^{-1}dx^2 - dy^2 - dz^2. \tag{3}$$

Counter Eamples

- Counterexamples: Schild, A. Equivalence Principle and Red-Shift Measurements. Am. J. Phys. 1960, 28, 778
- Counterexamples: Gruber R P, Price R H, Matthews S M, Cordwell W R and Wagner L F 1988 The impossibility of a simple derivation of the Schwarzschild metric Am. J. Phys. 56 265-269
- Counterexamples: Rindler W 1968 Counterexample to the Lenz-Schiff argument Am. J. Phys. 36 540-544 "
- Kassner; Impossible only with SR, EP and NL 2018
- "It is the spatial-distortion aspect of gravity that ensures that too simple a derivation of the Schwarzschild metric must fail". It is a coincidence Schwarzschild solution has recirpocal terms.

- In our analysis, we applied Einstein's equation.
- However, there could be a reason for the reciprocity condition.

$$ds^2 = [1 - \underline{v}^2]dt^2 - [1 - \underline{v}^2]^{-1}dx^2 - dy^2 - dz^2. \tag{4}$$

Plane Wave Describing Oscillations in Space and Time

Define a plane wave with oscillation in time and space:

$$t'_f = t' + t'_d = t' + \mathrm{Re}(\zeta_{t\mathbf{k}}) = t' + T_{\mathbf{k}} \sin(\mathbf{k} \cdot \mathbf{x}' - \omega t'), \tag{5}$$

$$\mathbf{x}'_f = \mathbf{x}' + \mathbf{x}'_d = \mathbf{x}' + \mathrm{Re}(\zeta_{\mathbf{x}\mathbf{k}}) = \mathbf{x}' + \mathbf{X}_{\mathbf{k}} \sin(\mathbf{k} \cdot \mathbf{x}' - \omega t'), \tag{6}$$

where

$$\zeta_{t\mathbf{k}} = -iT_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}' - \omega t')}, \tag{7}$$

$$\zeta_{\mathbf{x}\mathbf{k}} = -i\mathbf{X}_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}' - \omega t')}, \tag{8}$$

- $\zeta_{t\mathbf{k}}$ and $\zeta_{\mathbf{x}\mathbf{k}}$ form a Lorentz covariant plane wave

$$\begin{bmatrix} \zeta_{t\mathbf{k}} \\ \zeta_{\mathbf{x}\mathbf{k}} \end{bmatrix} = -i \begin{bmatrix} T_{\mathbf{k}} \\ \mathbf{X}_{\mathbf{k}} \end{bmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}' - \omega t')}. \tag{9}$$

Decomposition of Temporal Oscillation

- Proper time oscillation at $\mathbf{x}_0$ is a pulse that can be decomposed.
- For a relativistic theory, utilize Lorentz covariant plane waves for the decomposition

$$\begin{bmatrix} \bar{\xi}_{t\mathbf{k}} \\ \bar{\xi}_{\mathbf{x}\mathbf{k}} \end{bmatrix} = -i \begin{bmatrix} \bar{T}_{\mathbf{k}} \\ \bar{\mathbf{X}}_{\mathbf{k}} \end{bmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \quad (10)$$

- Plane waves to characterize the fluctuations in spacetime geometry caused by the proper time oscillation.

Decomposition of Temporal Oscillation

$$\begin{bmatrix} \bar{\xi}_{t\mathbf{k}} \\ \bar{\xi}_{\mathbf{xk}} \end{bmatrix} = -i \begin{bmatrix} \bar{T}_{\mathbf{k}} \\ \bar{\mathbf{X}}_{\mathbf{k}} \end{bmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \tag{11}$$

- Apply $\bar{\xi}_{t\mathbf{k}}$ from Eq. (11) to carry out the decomposition for the proper time oscillation.
- $\bar{\xi}_{t\mathbf{k}}$ is only the 0-component of a Lorentz covariant plane wave.
- Spatial component $\bar{\xi}_{\mathbf{xk}}$ cannot be neglected.
- Superpose $\bar{\xi}_{t\mathbf{k}}$ to obtain the proper time oscillation will have spatial oscillations associated with the superposition of $\bar{\xi}_{\mathbf{xk}}$.

Rest Mass System

In spherical coordinates, the proper time oscillation and the radial oscillations revealed after the superposition are:

At $r = 0$,

$$\bar{t}_f(t, 0) = t - \frac{\sin(\omega_0 t)}{\omega_0}, \tag{12}$$

$$\bar{r}_f(t, 0) = 0. \tag{13}$$

At $r = \epsilon/2 \to 0$,

$$\bar{t}_f(t, \epsilon/2) = t, \tag{14}$$

$$\bar{r}_f(t, \epsilon/2) = \epsilon/2 + \Re_\infty \cos(\omega_0 t), \tag{15}$$

where $\Re_\infty$ is the amplitude of radial oscillations.

Radial Oscillations not Vibration of Matter through Space

$$\bar{r}_f(t, \epsilon/2) = \epsilon/2 + \Re_\infty \cos(\omega_0 t), \tag{16}$$

- Radial oscillations results from superposing the spatial component of the Lorentz covariant plane waves.
- Radial oscillations oscillate about a thin shell Σ_0 with infinitesimal radius ($r = \epsilon/2 \to 0$).
- Amplitude of the radial oscillation ($\Re_\infty \to \infty$) violate relativity if oscillations involve motions of matter.
- Consider the radial oscillation as a spacetime geometrical effect acting on an observer stationary on the thin shell Σ_0.

Fictitious Frame of Oscillation

$$\bar{t}_f(t, \epsilon/2) = t, \tag{17}$$

$$\bar{r}_f(t, \epsilon/2) = \epsilon/2 + \Re_\infty \cos(\omega_0 t). \tag{18}$$

- Clock of O is synchronized with the clock of a 'fictitious' observer $\underline{O}$ that follows the radial oscillation defined in Eq. (18).
- Observer $\check{O}$ placed on the thin shell will oscillate relative to $\underline{O}$.
- Clocks of O and $\underline{O}$ are synchronized, the clocks of $\check{O}$ and O cannot be synchronized imply the spacetime geometry (or metrics) at O and $\check{O}$ are different.
- Study a thin shell with a finite radius that has an instantaneous fictitious velocity of less than the speed of light.

Thin Shell with Fictitious Radial Oscillations

- Investigate a similar timelike hypersurface Σ with finite radius $\check{r}$.
- Apply same fictitious oscillations but with instantaneous velocities $\bar{v}_f(t)$ less than the speed of light

$$\bar{t}_f(t, \check{r}) = t, \tag{19}$$

$$\bar{r}_f(t, \check{r}) = \check{r} + \Re \cos(\omega_0 t), \tag{20}$$

$$\bar{v}_f(t, \check{r}) = \frac{\partial \bar{r}_f(t, \check{r})}{\partial t} = -\Re\omega_0 \sin(\omega_0 t). \tag{21}$$

- Apply relativity to analyze the effects on the observer $\check{O}$ stationary on the thin shell's surface.

Time Translational Symmetry

- Fictitious oscillation has displacement and instantaneous velocity.
- Total energy generated by the instantaneous velocity and displacement constant over time for a harmonic oscillator.
- System has a time translational symmetry - Noether's theorem.
- Total effects of the instantaneous velocity and displacement on $\check{O}$ are constant over time.

Measurements on Thin Shell

- Relate infinitesimal coordinate increments (dt, dr) of two events observed by O in terms of the infinitesimal coordinate increments $(d\check{t}, d\check{r})$,

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} \Upsilon^{t}{}_{\check{t}} & 0 \\ 0 & \Upsilon^{r}{}_{\check{r}} \end{bmatrix} \begin{bmatrix} d\check{t} \\ d\check{r} \end{bmatrix}. \tag{22}$$

- Two off-diagonal terms of the transformation matrix Υ are zeros.
- Basis vectors of O and $\check{O}$ are parallel.
- Basis vectors in the temporal and spatial directions are orthogonal in the local frames of O and $\check{O}$.

Effects at $t = t_m = \pi/(2\omega_0)$

- At $t = t_m = \pi/(2\omega_0)$, fictitious displacement $\bar{r}_d(=\bar{r}_f - \breve{r})$ is zero.
- Instantaneous velocity is,

$$\bar{v}_f(t_m, \breve{r}) = \bar{v}_{fm} = -\Re\omega_0. \tag{23}$$

- $\breve{O}$ on the thin shell traveling at $\underline{v}_{fm}(= -\bar{v}_{fm} = \Re\omega_0 < 1)$ relative to $\underline{O}$ without displacement.
- Apply relativity to study the properties of a moving observer.
- At this instant, measurements by $\breve{O}$ undergo length contraction and time dilation relative to $\underline{O}$.

Time Translational Symmetry

- $\underline{O}$ is a fictitious inertial observer with its clock synchronized with O at spatial infinity.
- Although $\breve{O}$ remains stationary with O, its measurements will undergo the same length contraction and time dilation relative to O

$$\Upsilon^{t}{}_{\breve{t}} = [1 - (\bar{v}_{fm})^2]^{-1/2} = (1 - \Re^2\omega_0^2)^{-1/2}, \tag{24}$$

$$\Upsilon^{r}{}_{\breve{r}} = [1 - (\bar{v}_{fm})^2]^{1/2} = (1 - \Re^2\omega_0^2)^{1/2}. \tag{25}$$

- Based on time translational symmetry, results are cosntant over time

$$ds^2 = [1 - \Re^2\omega_0^2]dt^2 - [1 - \Re^2\omega_0^2]^{-1}dr^2 - \breve{r}^2 d\Omega^2. \tag{26}$$

Schwarzschild Field

Metric at $r = \breve{r}$ is line element of Schwarzschild if

$$m = \frac{\breve{r}\breve{\Re}^2\omega_0^2}{2}. \tag{27}$$

The vacuum space–time υ^+ outside this time-like hypersurface is the Schwarzschild spacetime,

$$ds^2 = [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]dt^2 - [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]^{-1}dr^2 - r^2 d\Omega^2. \tag{28}$$

Contraction of Thin Shell

- Time-like hypersurface Σ can be contracted per Birkhoff's theorem.
- As long as mass m of the shell is remaining constant, the metric and curvature of the external field will not be affected.
- The amplitude of the radial oscillation is,

$$\breve{\Re} = \sqrt{\frac{2}{\breve{r}\omega_0}}, \tag{29}$$

- Spacetime curvature tensors derived are well defined as the shell is contracted until it reaches a radius $\breve{r} = \epsilon/2$.
- Shell becomes infinitely small but with $\breve{\Re} \to \infty$.
- This infinitely small shell of radius $\breve{r} = \epsilon/2$ is the same shell we have described earlier.
- As predicted by Birkhoff's theorem, the metric around this infinitely small shell is the Schwarzschild spacetime.

Singularity

- When shell is contracted to a radius $\check{r} = 2m$ (the event horizon), the metric still encounters a coordinate singularity.
- Although fictitious instantaneous velocity on a shell inside event horizon can exceed the speed of light (i.e. $\underline{v}_{fm} > 1$ when $\check{r} < 2m$), they are not physical vibrations of matter.
- As information about the geometrical properties of spacetime, there is no superluminal transfer of energy.
- The metric on the surface of the shell is well defined until the radius is contracted to $\check{r} = \epsilon/2$.
- Shell can be contracted even beyond radius $\check{r} = 2m$ as allowed by Birkhoff's theorem while maintaining Schwarzschild geometry.

Gavitational Field

The proper time oscillator exerts fictitious radial oscillations on a thin shell with an infinitesimal radius. These radial oscillations alter the spacetime metric on the thin shell's surface and curve the surrounding external spacetime. In turn, the curved spacetime tells other matter how to react in the presence of the proper time oscillator.

Proper time oscillaton matter field must be quantized

Plane Wave Describing Oscillations in Space and Time

Define a plane wave with oscillation in time and space:

$$t'_f = t' + t'_d = t' + \mathrm{Re}(\zeta_{t\mathbf{k}}) = t' + T_{\mathbf{k}} \sin(\mathbf{k} \cdot \mathbf{x}' - \omega t'), \tag{30}$$

$$\mathbf{x}'_f = \mathbf{x}' + \mathbf{x}'_d = \mathbf{x}' + \mathrm{Re}(\zeta_{\mathbf{x}\mathbf{k}}) = \mathbf{x}' + \mathbf{X}_{\mathbf{k}} \sin(\mathbf{k} \cdot \mathbf{x}' - \omega t'), \tag{31}$$

where

$$\zeta_{t\mathbf{k}} = -iT_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}' - \omega t')}, \tag{32}$$

$$\zeta_{\mathbf{x}\mathbf{k}} = -i\mathbf{X}_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}' - \omega t')}, \tag{33}$$

- $\zeta_{t\mathbf{k}}$ and $\zeta_{\mathbf{x}\mathbf{k}}$ form a Lorentz covariant plane wave

$$\begin{bmatrix} \zeta_{t\mathbf{k}} \\ \zeta_{\mathbf{x}\mathbf{k}} \end{bmatrix} = -i \begin{bmatrix} T_{\mathbf{k}} \\ \mathbf{X}_{\mathbf{k}} \end{bmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}' - \omega t')}. \tag{34}$$

Plane Wave Describing Oscillations in Space and Time

Define a plane wave,

$$\zeta_{\mathbf{k}} = \frac{T_{0\mathbf{k}}}{\omega_0} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \tag{35}$$

Temporal and spatial oscillation displacements can be written as

$$\zeta_{t\mathbf{k}} = \partial_0 \zeta_{\mathbf{k}} = -iT_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \tag{36}$$

$$\zeta_{\mathbf{x}\mathbf{k}} = -\nabla \zeta_{\mathbf{k}} = -i\mathbf{X}_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \tag{37}$$

$\zeta_{\mathbf{k}}$ satisfies the Klein Gordon equation:

$$\partial_u \partial^u \zeta_{\mathbf{k}} + \omega_0^2 \zeta_{\mathbf{k}} = 0. \tag{38}$$

Hamiltonian Density

- A system in a volume V with multiple particles of mass m.
- Impose periodic boundary conditions at the box walls.
- The corresponding Hamiltonian density

$$\mathcal{H}_{\mathbf{k}} = \frac{m\omega_0^2}{2V}[(\partial_0\zeta_{\mathbf{k}}^*)(\partial_0\zeta_{\mathbf{k}}) + (\nabla\zeta_{\mathbf{k}}^*)\cdot(\nabla\zeta_{\mathbf{k}}) + \omega_0^2\zeta_{\mathbf{k}}^*\zeta_{\mathbf{k}}]. \tag{39}$$

Proper Time Quantization

- Consider a plane wave with oscillation in proper time only

$$\zeta_0 = \frac{T_0}{\omega_0} e^{-i\omega_0 t}. \tag{40}$$

- Hamiltonian density is,

$$\mathcal{H}_0 = (\frac{m\omega_0^2}{V}) T_0^* T_0. \tag{41}$$

- Matter in plane wave has no spatial motion.
- Energy belong to some intrinsic energy of the system.
- Field considering is 'free' with no charges or force fields.
- Adopt the energy in $\mathcal{H}_0$ as the intrinsic mass-energy of matter.

- Plane wave with n number of particles, Hamiltonian density is $\mathcal{H}_0 = nm/V$.
- Compare with Eq. (41), energy E in volume V is,

$$E = nm = m\omega_0^2 T_0^* T_0, \tag{42}$$

- Mass is on-shell leads to a quantization condition,

$$\omega_0^2 T_0^* T_0 = n. \tag{43}$$

- Number of particles is discrete - **Proper Time Oscillators**.
- Matter field with proper time oscillations is a **quantized field**.

Bosonic Field

Matter field with oscillations in time has same properties of a bosonic field

Bosonic Field

- A real scalar field by superposition of $\zeta_{\mathbf{k}}$ and $\zeta_{\mathbf{k}}^*$

$$\zeta(x) = \sum_{\mathbf{k}} (2\omega\omega_0)^{-1/2} [T_{0\mathbf{k}} e^{-ikx} + T_{0\mathbf{k}}^* e^{ikx}]. \quad (44)$$

- Transform into a quantized field through canonical quantization.
- Relate $\zeta(x)$ with the bosonic field $\varphi(x)$ in quantum theory

$$\varphi(x) = \zeta(x)\sqrt{\frac{\omega_0^3}{V}} = \sum_{\mathbf{k}} (2\omega V)^{-1/2} [a_{\mathbf{k}} e^{-ikx} + a_{\mathbf{k}}^\dagger e^{ikx}]. \quad (45)$$

- Annihilation and creation operators

$$a_{\mathbf{k}} = \omega_0 T_{0\mathbf{k}}, \quad a_{\mathbf{k}}^\dagger = \omega_0 T_{0\mathbf{k}}^\dagger. \quad (46)$$

- **Matter field with oscillations in time has same properties of a bosonic field**.

Internal time is a self-adjoint operator

Self-Adjoint Time Operator

- Displaced time linearly related to the conjugate momenta $\eta(x)$

$$t_d(x) = \zeta_t(x) = \partial_0\zeta(x) = \sum_{\mathbf{k}} \frac{-i}{\sqrt{2}}[\tilde{T}_{\mathbf{k}} e^{-ikx} - \tilde{T}^{\dagger}_{\mathbf{k}} e^{ikx}] = \frac{\eta(x)V}{\omega_0^3}. \quad (47)$$

- $t_d(x)$ and $\zeta(x)$ also form a conjugate pair

$$(\frac{\omega_0^3}{V})[\zeta(t,\mathbf{x}), t_d(t,\mathbf{x}')] = i\delta(\mathbf{x}-\mathbf{x}'), \quad (48)$$

$$[t_d(t,\mathbf{x}), t_d(t,\mathbf{x}')] = 0. \quad (49)$$

- $\zeta(x)$, $\eta(x)$ and $t_d(x)$ are self-adjoint operators.
- Displaced time t_d oscillates back and forth relative to the external time $t$and its spectrum is not bounded.

Self-Adjoint Time Operator

- Internal time in a matter field is

$$t_f(t, \mathbf{x}) = t + t_d(t, \mathbf{x}). \qquad (50)$$

- t is a parameter, but $t_d(t, \mathbf{x})$ is a self-adjoint operator.
- Internal time t_f must also be a **self-adjoint operator.**
- **No conflict with Pauli's theorem.**

Proper time uncertainty relation analogous to position-momentum uncertainty relation

Proper Time Field

Consider a real scalar field that has oscillations of matter in proper time only

$$\zeta' = \frac{1}{\sqrt{2}}[\zeta_0 + \zeta_0^\dagger] = \frac{1}{\sqrt{2}\omega_0}[T_0 e^{-i\omega_0 t} + T_0^\dagger e^{i\omega_0 t}]. \tag{51}$$

- Displaced time t'_d and displaced time rate u'_d are,

$$t'_d = \frac{-i}{\sqrt{2}}[T_0 e^{-i\omega_0 t} - T_0^\dagger e^{i\omega_0 t}] = \frac{-i}{\sqrt{2}\omega_0}[ae^{-i\omega_0 t} - a^\dagger e^{i\omega_0 t}], \tag{52}$$

$$u'_d = \partial_0 t'_d = \frac{-\omega_0}{\sqrt{2}}[T_0 e^{-i\omega_0 t} + T_0^\dagger e^{i\omega_0 t}] = \frac{-1}{\sqrt{2}}[ae^{-i\omega_0 t} + a^\dagger e^{i\omega_0 t}]. \tag{53}$$

- The Hamiltonian density is

$$H' = \frac{1}{2}(m\omega_0^2 {t'_d}^2 + \frac{{P'_d}^2}{m}) = \omega_0(a^\dagger a + \frac{1}{2}), \tag{54}$$

where

$$P'_d = mu'_d. \tag{55}$$

Uncertainty Relations Comparison

Table 1		
	Proper Time Oscillator	**Quantum Harmonic Oscillator**
Hamiltonian	$H' = \omega_0(a^\dagger a + \frac{1}{2})$	$H = \omega(a^\dagger a + \frac{1}{2})$
Commutation Relation	$[t'_d, P'_d] = i$	[x,p]=i
Uncertainty Relation	$\Delta t'_d \Delta P'_d \geq \frac{1}{2}$	$\Delta x \Delta p \geq \frac{1}{2}$

Creation and annihilation operators for bosonic field and quantum harmonic oscillator have similar formulation. Can there be a hidden symmetry?

Fermionic Field with Proper Time Oscillation

Dirac Field

As an intrinsic property of a particle, the properties of mass are the same for all massive particles regardless of their spins.

$$\psi_\alpha = \frac{1}{\sqrt{V}} \sum_s \sum_p \frac{1}{\sqrt{2E_p}} (\omega_0 T_{0a}(p,s) u_\alpha(p,s) e^{-ip\dot{x}} + \omega_0 T_{0b}^\dagger(p,s) v_\alpha(p,s) e^{ip\dot{x}}), \tag{56}$$

$$\psi_\alpha^\dagger = \frac{1}{\sqrt{V}} \sum_s \sum_p \frac{1}{\sqrt{2E_p}} (\omega_0 T_{0a}\dagger(p,s) u_\alpha^\dagger(p,s) e^{-ip\dot{x}} + \omega_0 T_{0b}(p,s) v_\alpha^\dagger(p,s) e^{ip\dot{x}} \tag{57}$$

Rewrite creation and annihilition operators for fermion and anti-fermion in terms of the proper time amplitudes:

$$a = \omega_0 T_{0a} \quad b^\dagger = \omega_0 T_{0b}^\dagger. \tag{58}$$

Dirac Field

Proper time amplitudes satisfy anti-commutation relations:

$$\{T_{0a}(\mathbf{p},s),\, T_{0a}^{\dagger}(\mathbf{p}'s')\} = \delta_{ss'}\delta_{\mathbf{pp}'}/\omega_0^2, \tag{59}$$

$$\{T_{0b}(\mathbf{p},s),\, T_{0b}^{\dagger}(\mathbf{p}'s')\} = \delta_{ss'}\delta_{\mathbf{pp}'}/\omega_0^2. \tag{60}$$

The Hamiltion is:

$$H = \sum_{\mathbf{p},s} E(p)\omega_0^2[T_{0a}^{\dagger}(p.s)T_{0a}(p,s) + T_{0b}^{\dagger}(p,s)T_{0b}(p,s)], \tag{61}$$

which are the summation of energy for fermions and anti-fermions with proper time oscillations.

Electromagnetic Field with Proper Time Oscillation

Electromagnetic Field

Electromagmetic field with polarization $\lambda = 1, 2$,

$$A_\mu = \int \frac{d^3p}{2p_0(2\pi)^3} \sum_\lambda [a_{\lambda,p}\epsilon_p^{(\lambda)} e^{-ip\dot{x}} + a^\dagger_{\lambda,p}(\epsilon_p^{(\lambda)})^\dagger e^{ip\dot{x}}]. \quad (62)$$

- Electromagnetic field has same structure as the fermionic field but with polarization.
- Photon has no proper time! Whether a photon has oscillation can only be determined by experiments.
- What happen after spontaneous symmetry breaking?

Higgs Boson as Proper Time Oscillator

Higgs Field

- **Higgs Boson as a particle with oscillation in proper time.**
- Beginning with a massless gauge field A_μ and a complex scalar field $\phi = \phi_1 + i\phi_2$.
- The Lagrangian of this photon field coupling to a scalar field assumes the form,

$$L = -\frac{1}{4}F_{\mu\upsilon}F^{\mu\upsilon} + (D_\mu\phi)^*(D^\mu\phi) - V(\phi^2). \quad (63)$$

where

$$V(\phi^2) = \mu^2\phi^2 + \lambda\phi^4. \quad (64)$$

Higgs Field

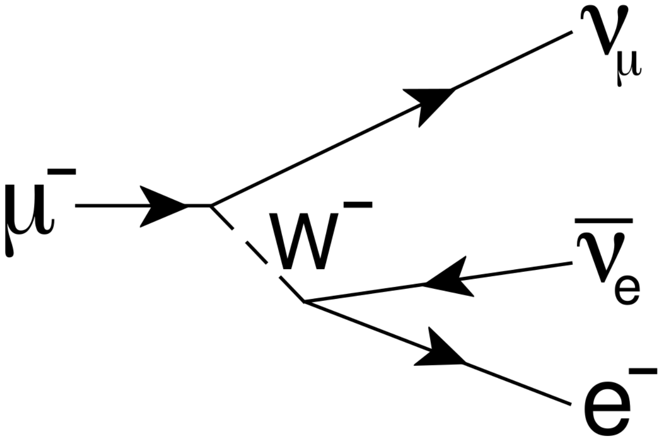
- Choosing the ground state at $\phi = \upsilon$ spontaneously break the symmetry of the Lagrangian.
- The resulting Lagrangian density is

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h - \lambda \upsilon^2 h^2 - \frac{1}{4} F_{\mu\upsilon} F^{\mu\upsilon} + \frac{1}{2} g_0^2 \upsilon^2 A_\mu A^\mu + g_0^2 \upsilon A_\mu A^\mu h + \frac{1}{2} g_0^2 A_\mu A^\mu h^2 - \lambda \upsilon h^3 - \frac{1}{4}\lambda h^4.$$

- The Higgs field $h(x)$ has mass $\sqrt{2\lambda}\upsilon$. The photon in an Abelian gauge field acquires a mass $m = g_0 \upsilon$.
- **Interaction with Higgs field causes some of the massless bosons to oscillate in proper time.**

Consistent with the predictions of quantum theory and general relativity

Muon Decay Time



Muon Decay Time

The uncertainty of decay time measurement.

$$\Delta t' = \sqrt{\frac{\omega}{2\omega_0^3}} = \hbar\sqrt{\frac{E}{2m^3}}. \tag{65}$$

- Muon mass-energy $m_\mu = 105.6583744 \times 10^6$ eV.
- Assume projected energy $E = 1$TeV.
- Uncertainty $\Delta t' = 4.3 \times 10^{-22}$s.
- Mean life time of muon decay $\Delta t_\mu = 2.1969811(22) \times 10^{-6}$s

Moving Oscillator

- Consider a normalized plane wave

$$\tilde{\zeta} = \frac{e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}}{\sqrt{\omega\omega_0^3}}. \tag{66}$$

- Hamiltonian density is

$$\tilde{\mathcal{H}} = \omega/V. \tag{67}$$

- Observed particle travels at an average velocity of $\mathbf{v} = \mathbf{k}/\omega$.
- As the particle propagates, it oscillates with amplitudes

$$\mathring{T} = \sqrt{\frac{\omega}{\omega_0^3}}, \quad \mathring{\mathbf{X}} = \frac{\mathbf{k}}{\sqrt{\omega_0^3\omega}}. \tag{68}$$

- **At a higher energy level, the effects of the particle's oscillations will be easier to detect.**

Moving Oscillator

Even for a π^+ particle with an energy 1 TeV, detecting the oscillations is still beyond the reach of our experiments.

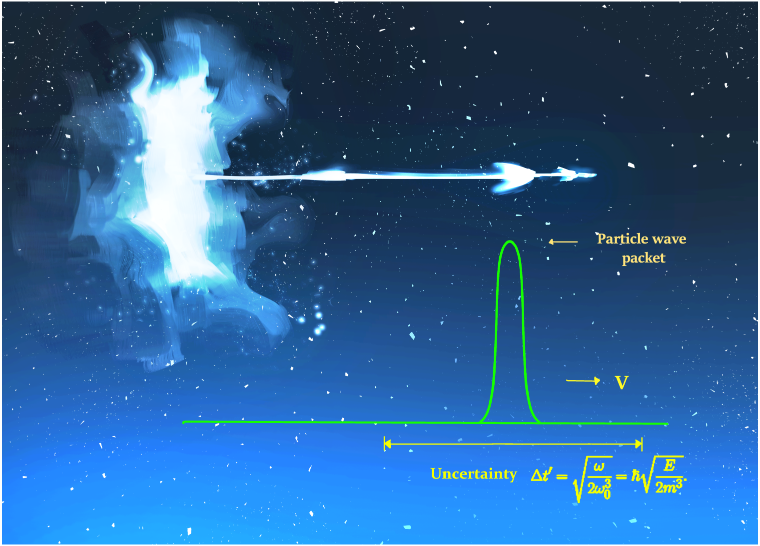
Table 1 Oscillation amplitudes of a π^+ with different projected energies.

$E(GeV)$	$\mathring{T}(s)$	$\mathring{X}(m)$
1	1.3×10^{-23}	3.5×10^{-15}
10	4.0×10^{-23}	1.2×10^{-14}
100	1.3×10^{-22}	3.8×10^{-14}
1000	4.0×10^{-22}	1.2×10^{-13}

Consistancy with Standard Theory

Poperties of a proper time oscillator stay consistent with the predictions of quantum theory until we reach a very high energy level, where the oscillations of matter in time and space become significant. However, the oscillations are small and cannot be detected by experiments yet.

Neutrino's arrival time



Particle wave packet
V
Uncertainty $\Delta t' = \sqrt{\frac{\omega}{2\omega_0^3}} = \hbar\sqrt{\frac{E}{2m^3}}$.

Particle's Arrival Time

Neutrino has extreme small mass and much larger amplitudes of oscillations.

$E(GeV)$	$\mathring{T}(s)$	$\mathring{X}(cm)$	$\omega_p(s^{-1})$
1	7.4×10^{-12}	0.22	6.1×10^{6}
10	2.3×10^{-11}	0.70	6.1×10^{5}
100	7.4×10^{-11}	2.20	6.1×10^{4}
1000	2.3×10^{-10}	7.00	6.1×10^{3}

Note: The assumed mass of the particle is $m = 2eV$.

Estimate for Neutrio Mass

The deviations will result in an uncertainty of arrival time when we measure a large collection of particles with the same average velocity, i.e.

$$\Delta t' = \sqrt{\frac{\omega}{2\omega_0^3}} = \hbar\sqrt{\frac{E}{2m^3}}. \tag{69}$$

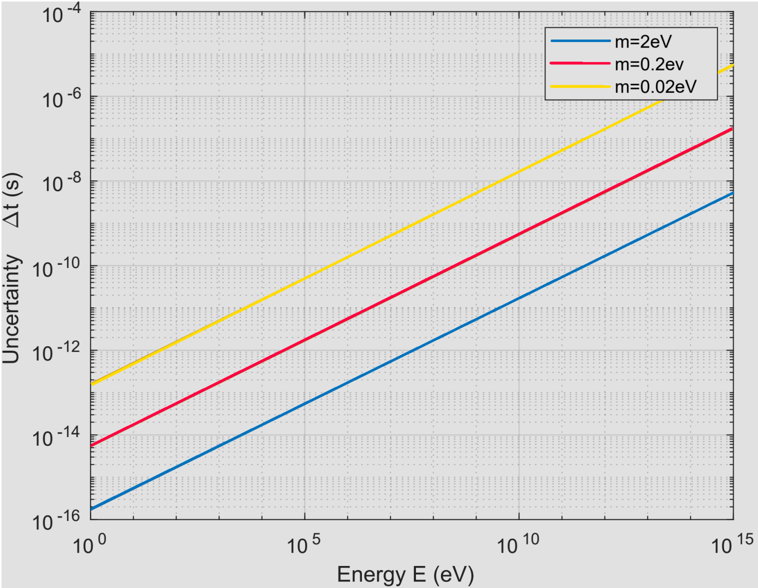
With the arrival time uncertainty obtained from experiments, the mass of a neutrino can be reconciled,

$$m = [\frac{\hbar^2 E}{2(\Delta t')^2}]^{1/3}. \tag{70}$$

Fluctuation of Neutrino's Arrival Time

- The experiments (e.g. IceCube) on neutrinos' speed could provide some hints.
- Study in lightcone fluctuation evaluated the accumulated uncertainty effects of a neutrino's travel time and distance in fluctuating spacetime.
- Suggested uncertainty follows a power-law depending on the neutrino's energy, i.e., $\Delta t' \propto l^m E^n$. , where m and n are factors to be established by experiments or theoretical predictions.
- **Uncertainty derived from temporal oscillation is $\Delta t' \propto E^{1/2}$ akin to the power-law in quantum spacetime.**
- Assuming $m = 0.2eV$ and $E = 1TeV$, uncertainty is in the order of $10^{-9}s$.

Fluctuation of Neutrino's Arrival Time



Conclusion

Assuming matter can oscillate in proper time:

- Reconcile basic properties of a quantum field.
- Spacetime around has the Schwarzschild solution.
- Neutrino time of arrival may provide hints.

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