The Relativistic Virial Theorem

A Possible Resolution of the "Dark Matter" Problem up to the Galactic Cluster Scale

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Preface

ARIEL UNIVERSITY

William of Ockham
1285 - 1347

Ockham's Razor:

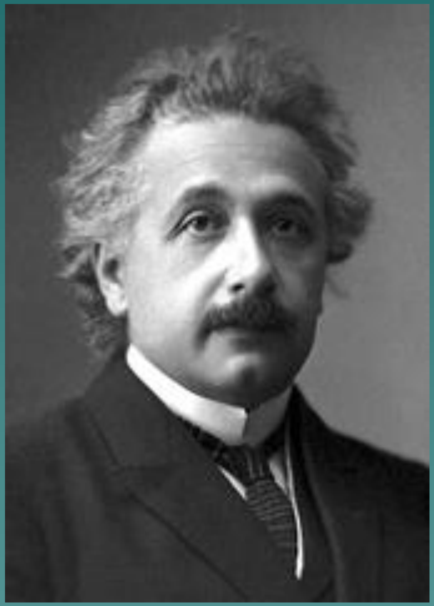
"Plurality is not to be posited without necessity."

Don't multiply complex causes to explain things when a simple one will do

Law of Parsimony (lex parsimoniae).

Albert Einstein

1879 - 1955



"It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience."

"On the Method of Theoretical Physics" the Herbert Spencer Lecture, Oxford, June 10,1933.

Dark Matter is not allowed in science unless it is necessary

Observational Status

Extensive, now 40-year underground and accelerator searches have failed to find any dark matter or establish its existence. The dark matter situation has become even more dire in the last few years as the Large Hadron Collider has failed to find any super symmetric particles, not only of the community’s preferred form of dark matter, but also the form of it that is required in string theory, a theory that attempts to provide a quantized version of Newton–Einstein gravity.

Retardation

Galaxy clusters (but also galaxies) are huge physical systems having dimensions of tens of millions of light years. Thus, any change at the cluster center will be noticed at the rim only tens of millions of years later. Those retardation effects seems to be neglected in naïve cluster modelling used to calculate the dispersion of velocities in the cluster.

Retardation Theory & Weak Field GR

General Relativity

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$T_{\mu\nu} = (p + \rho c^2) u_\mu u_\nu - p\, g_{\mu\nu}$$

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = \frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} u^\mu u^\nu = 0$$

General Relativity

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R.$$

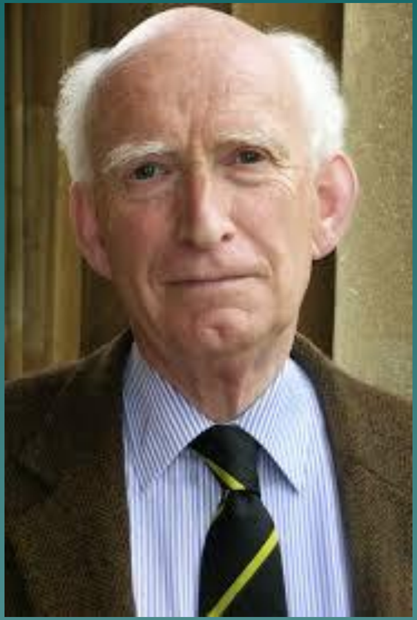
$$R^{\mu}_{\nu\alpha\beta} = \Gamma^{\mu}_{\nu\alpha,\beta} - \Gamma^{\mu}_{\nu\beta,\alpha} + \Gamma^{\sigma}_{\nu\alpha}\Gamma^{\mu}_{\sigma\beta} - \Gamma^{\sigma}_{\nu\beta}\Gamma^{\mu}_{\sigma\alpha}, \quad R_{\alpha\beta} = R^{\mu}_{\alpha\beta\mu}, \quad R = g^{\alpha\beta} R_{\alpha\beta}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta} \right), \qquad g_{\beta\mu,\nu} \equiv \frac{\partial g_{\beta\mu}}{\partial x^{\nu}}$$

How can one solve those complex, tensor, non-linear partial differential equations for the case of galaxies?

Answer:

- For most cases (galaxy cluster included) it is not necessary to solve the full Einstein equation but only a linear approximation to them as only weak gravitational fields are involved.
- For some cases such as compact objects (black holes) and the very early universe (big bang) strong gravitational fields are involved, and one needs to use the exact Einstein equations.



5 April 1935 – 6 February 2018

I would like to thank the late Professor Donald Lynden-Bell for this very important observation.

Linear Approximation (weak gravity)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \eta_{\mu\nu} \equiv \text{ diag } (1, -1, -1, -1), \quad |h_{\mu\nu}| \ll 1$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h, \quad h = \eta^{\mu\nu} h_{\mu\nu}.$$

$$\Box \bar{h}_{\mu\nu} \equiv \bar{h}_{\mu\nu,\alpha}{}^{\alpha} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \qquad \bar{h}_{\mu\alpha,}{}^{\alpha} = 0.$$

Solution:

$$\bar{h}_{\mu\nu}(\vec{x}, t) = -\frac{4G}{c^4}\int \frac{T_{\mu\nu}(\vec{x}', t - \frac{R}{c})}{R} d^3x',$$

$$t \equiv \frac{x^0}{c}, \quad \vec{x} \equiv x^a \quad a, b \in [1, 2, 3], \quad \vec{R} \equiv \vec{x} - \vec{x}', \quad R = |\vec{R}|.$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}.$$

$$\frac{4G}{c^4} \simeq 3.3 \ 10^{-44}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\eta^{\alpha\beta}\left(h_{\beta\mu,\nu} + h_{\beta\nu,\mu} - h_{\mu\nu,\beta}\right).$$

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = \frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu}u^\mu u^\nu = 0$$

Affine connection is first order.

$u^\mu u^\nu$ zeroth order

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu,$$

$$u^0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad u^a = \vec{u} = \frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad \vec{v} \equiv \frac{d\vec{x}}{dt}, \quad v = |\vec{v}|.$$

$$u^0 \simeq 1, \qquad \vec{u} \simeq \frac{\vec{v}}{c}, \qquad u^a \ll u^0 \qquad \text{for} \quad v \ll c.$$

The resulting geodesic:

$$\frac{dv^a}{dt} \simeq -c^2\Gamma^a_{00} = -c^2\left(h^a_{0,0} - \frac{1}{2}h_{00,}{}^a\right)$$

$$\rho c^2 \gg p$$

$$\frac{dv^a}{dt} \simeq \frac{c^2}{4}\bar{h}_{00,}{}^a \Rightarrow \frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}, \qquad \phi \equiv \frac{c^2}{4}\bar{h}_{00}$$

Back to Newton?

$$\phi = \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' = -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x'$$

If retardation is neglected or the density is static:

$$\phi = \phi_N = -G\int \frac{\rho(\vec{x}')}{R} d^3x'$$

Retarded Gravity ≠ Retarded Newtonian Force

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}$$

$$\begin{aligned} \phi &= \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' \\ &= -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x' \end{aligned}$$

The force is always partitioned to a retarded Newtonian force and a retardation force that has no parallel in Newtonian theory.

$$\vec{F} = \vec{F}_{Nr} + \vec{F}_r$$

$$\vec{F}_{Nr} = -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R^2}\hat{R}d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R}$$

$$\vec{F}_r \equiv -\frac{G}{c}\int \frac{\rho^{(1)}(\vec{x}', t - \frac{R}{c})}{R}\hat{R}d^3x', \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}.$$

We suggest that retardation cannot be neglected on galaxy cluster scales and the density is not static.

Beyond the Newtonian Approximation

$$\rho(\vec{x}', t - \frac{R}{c}) = \sum_{n=0}^{\infty} \frac{1}{n!} \rho^{(n)}(\vec{x}', t)(-\frac{R}{c})^n, \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}.$$

$$\rho(\vec{x}', t - \frac{R}{c}) \simeq \rho(\vec{x}', t) - \rho^{(1)}(\vec{x}', t)\frac{R}{c} + \frac{1}{2}\rho^{(2)}(\vec{x}', t)(\frac{R}{c})^2.$$

A Taylor series expansion has a limited range hence the current approach is "near field" that is of limited distance Validity from the cluster.

$$\phi = -G\int \frac{\rho(\vec{x}',t)}{R} d^3x' + \frac{G}{c}\int \rho^{(1)}(\vec{x}',t)d^3x' - \frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}',t)d^3x'$$

$$\phi_r = -\frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}',t)d^3x'$$

$$\begin{aligned}
\vec{F} &= \vec{F}_N + \vec{F}_r \\
\vec{F}_N &= -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}',t)}{R^2}\hat{R}d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R} \\
\vec{F}_r &\equiv -\vec{\nabla}\phi_r = \frac{G}{2c^2}\int \rho^{(2)}(\vec{x}',t)\hat{R}d^3x'
\end{aligned}$$

Obviously for small distances Newtonian forces dominate over Retardation forces but what happens for large distances were Newtonian forces decline and Retardation forces are not reduced?

Δt is the typical duration in which the density ρ changes.

$$R_r \equiv c\Delta t$$

Retardation distance

$$R \ll R_r$$

Newtonian regime

$$R \gg R_r$$

Retardation regime

This statement will be made more precise later.

The virial theorem and the Comma Cluster

The origin of “dark matter”

The Swiss-American astronomer Fritz Zwicky is arguably the most famous and widely cited pioneer in the field of dark matter.

He studied (1933) the redshifts of various galaxy clusters, as published by Hubble and Humason, and noticed a large scatter in the apparent velocities of eight galaxies within the Coma Cluster, with differences that exceeded 2000 km/s.

The fact that Coma exhibited a large velocity dispersion with respect to other clusters had already been noticed by Hubble and Humason, but Zwicky went a step further, applying the virial theorem to the cluster in order to estimate its mass.

Zwicky started by estimating the total mass of Coma to be the product of the number of observed galaxies, 800 (Current estimates is more than 1000), and the average mass of a galaxy, which he took to be 10^9 solar masses, as suggested by Hubble. He then adopted an estimate for the physical size of the system, which he took to be around 10^6 light years (a more recent estimate is a diameter larger than 10^7 light years), in order to determine the potential energy of the system.

From there, he calculated the average kinetic energy and finally a velocity dispersion. He found that 800 galaxies of 10^9 solar masses in a sphere of 10^6 light years in diameter should exhibit a velocity dispersion of 80 km/s.

In contrast, the observed average velocity dispersion along the line of sight was approximately 1000 km/s. From this comparison, he concluded the following:

"If this would be confirmed, we would get the surprising result that dark matter is present in much greater amount than luminous matter."

This sentence is sometimes cited in the literature as the first usage of the phrase dark matter.

In 1937, Zwicky published a new article in the Astrophysical Journal, in which he refined and extended his analysis of the Coma Cluster.

The purpose of this paper was to determine the mass of galaxies. In particular, he returned to the virial theorem approach that he had proposed in 1933, this time assuming that Coma contained 1000 galaxies within a radius of $2 \cdot 10^6$ light years and solving for the average galaxy's mass. From the observed velocity dispersion of 700 km/s, he obtained a conservative lower limit of $4.5 \cdot 10^{13}$ $M_\odot$ on the mass of the cluster (to be conservative, he excluded a galaxy with a recession velocity of 5100 km/s as a possible outlier), corresponding to an average mass per galaxy of $4.5 \cdot 10^{10}$ $M_\odot$.

Assuming then an average absolute luminosity for cluster galaxies of $8.5 \cdot 10^7$ times that of the Sun, Zwicky showed that this led to a surprisingly high mass-to-light ratio of about 500 $\frac{M_\odot}{L_\odot}$

Zwicky's work relied on Hubble's relationship between redshift and distance, and in the 1937 paper he used the results of Hubble and Humason, which pointed to a Hubble constant of H_0= 558 km/s/Mpc, with an estimated uncertainty of 10-20%. If we rescale these results adopting the modern value of H_0 = 67.27 \pm 0.66 km/s/Mpc, we see that Zwicky overestimated the distance by a factor of 558/67.27 = 8.3. Despite this substantial correction, Coma's velocity dispersion still implies a very high mass to light ratio and points to the existence of dark matter in some form provided that the virial theorem is correct for gravitating systems.

A crucial difference between Newtonian and Retarded Forces

$$\vec{f} = \vec{f}_N + \vec{f}_r$$

$$\vec{f}_N \equiv -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}', t)}{R^2}\hat{R}d^3x', \quad \hat{R} \equiv \frac{\vec{R}}{R},$$

$$\vec{f}_r \equiv -\vec{\nabla}\phi_r = -\frac{G}{2c^2}\int \rho^{(2)}(\vec{x}', t)\hat{R}d^3x'$$

Let us look at a point particle and calculate its Newtonian potential:

$$\rho_j = m_j \delta^{(3)}(\vec{x}' - \vec{r}_j(t))$$

$$\phi_{Nj} = -G\frac{m_j}{R_j(t)}, \qquad \vec{R}_j(t) = \vec{x} - \vec{r}_j(t), \qquad R_j(t) = |\vec{R}_j(t)|$$

And its retarded potential:

$$\phi_{rj} = -\frac{G m_j}{2c^2}\frac{\partial^2}{\partial t^2} R_j(t) = \frac{G m_j}{2c^2}\left(\hat{R}_j \cdot \vec{a}_j - \frac{\vec{v}_j^2 - (\vec{v}_j \cdot \hat{R}_j)^2}{R_j(t)}\right),$$

$$\hat{R}_j \equiv \frac{\vec{R}_j}{R_j}, \; \vec{v}_j \equiv \frac{d\vec{r}_j}{dt}, \; \vec{a}_j \equiv \frac{d\vec{v}_j}{dt}.$$

The gravitational force generated by particle j on particle k for both:

$$\begin{aligned}
\vec{F}_{j,k} &= \vec{F}_{Nj,k} + \vec{F}_{rj,k} \\
\vec{F}_{Nj,k} &= -G\frac{m_j m_k}{R_{k,j}^2}\hat{R}_{k,j}, \qquad \vec{R}_{k,j} \equiv \vec{r}_k - \vec{r}_j,\ R_{k,j} \equiv |\vec{R}_{k,j}(t)|,\ \hat{R}_{k,j} \equiv \frac{\vec{R}_{k,j}}{R_{k,j}}, \\
\vec{F}_{rj,k} &= \frac{G m_j m_k}{2R_{k,j}^2 c^2}\left(R_{k,j}\vec{a}_{\perp j,k} + \hat{R}_{k,j}\vec{v}_{\perp j,k}^2 - 2(\vec{v}_{j,k}\cdot\hat{R}_{k,j})\vec{v}_{\perp j,k}\right) \\
\vec{a}_{\perp j,k} &\equiv \vec{a}_j - (\vec{a}_j\cdot\hat{R}_{k,j})\hat{R}_{k,j}, \qquad \vec{v}_{\perp j,k} \equiv \vec{v}_j - (\vec{v}_j\cdot\hat{R}_{k,j})\hat{R}_{k,j}.
\end{aligned}$$

Newton's third law

$$\vec{F}_{N\,k,j} = -\vec{F}_{N\,j,k}$$

$$\vec{F}_{r\,k,j} \neq -\vec{F}_{r\,j,k}$$

Acceleration and velocity of particles j and k are unrelated.

The Virial Theorem – Newtonian Version

$$\bar{G} \equiv \sum_{k=1}^{N} \vec{p}_k \cdot \vec{r}_k, \qquad \vec{p}_k \equiv m_k \vec{v}_k.$$

$$\frac{d\bar{G}}{dt} = 2T + \sum_{k=1}^{N} \vec{F}_k \cdot \vec{r}_k, \qquad T \equiv \frac{1}{2}\sum_{k=1}^{N} m_k \vec{v}_k^2.$$

The Virial Theorem – Newtonian Version

$$\vec{F}_k \equiv \sum_{j=1}^{N} F_{j,k} \;\Rightarrow\; \sum_{k=1}^{N} \vec{F}_k \cdot \vec{r}_k = \sum_{k=1}^{N} \sum_{j=1}^{N} \vec{F}_{j,k} \cdot \vec{r}_k = \sum_{k=2}^{N} \sum_{j=1}^{k-1} (\vec{F}_{j,k} \cdot \vec{r}_k + \vec{F}_{k,j} \cdot \vec{r}_j),$$

$$\vec{F}_{j,k} = -\vec{F}_{k,j}$$

For Newton's gravity!

$$\Rightarrow \sum_{k=1}^{N} \vec{F}_k \cdot \vec{r}_k = \sum_{k=2}^{N} \sum_{j=1}^{k-1} \vec{F}_{j,k} \cdot \vec{R}_{k,j}.$$

The Virial Theorem – Newtonian Version

$$V_{Njk} = m_k \phi_{Nj} = m_j \phi_{Nk} = -\frac{G m_j m_k}{R_{k,j}} \Rightarrow \vec{F}_{j,k} = -\vec{\nabla}_{\vec{r_k}} V_{Njk} = -\frac{dV_{Njk}}{dR_{k,j}} \hat{R}_{k,j}$$

$$\sum_{k=1}^{N} \vec{F}_k \cdot \vec{r}_k = -\sum_{k=2}^{N} \sum_{j=1}^{k-1} \frac{dV_{Njk}}{dR_{k,j}} R_{k,j} = \sum_{k=2}^{N} \sum_{j=1}^{k-1} V_{Njk} = V_{NT}$$

The Virial Theorem – Newtonian Version

$$\frac{d\bar{G}}{dt} = 2T + V_{NT}.$$

The Virial Theorem – Newtonian Version

Taking a temporal average of the above quantity and making the reasonable assumption that for a bounded system $\bar{G}$ is always finite we arrive at the result:

$$\left\langle \frac{d\bar{G}}{dt} \right\rangle \equiv \lim_{\tau->\infty} \frac{1}{\tau} \int_0^\tau \frac{d\bar{G}}{dt} = \lim_{\tau->\infty} \frac{G(\tau)-G(0)}{\tau} = 0. \Rightarrow 2\langle T \rangle = -\langle V_{NT} \rangle = \langle |V_{NT}| \rangle .$$

An "average" velocity square is thus:

$$\left\langle v^2 \right\rangle_t \equiv \frac{1}{M} \left\langle \sum_{k=1}^{N} m_k \vec{v}_k^2 \right\rangle = 2\frac{\langle T \rangle}{M}, \qquad M = \sum_{k=1}^{N} m_k .$$

The Virial Theorem – Newtonian Version

Define the gravitational radius:

$$r_g = \frac{GM^2}{|V_{NT}|}$$

The Virial Theorem – Newtonian Version

$$\left\langle v^2 \right\rangle_t = \frac{\langle |V_{NT}| \rangle}{M} = \frac{GM}{r_g},$$

The gravitational radius was found to be closely relate to the median radius r_h in which half the systems mass is contained such that $r_h \simeq 0.4 r_g$

Binney, J.; & Tremaine, S. *Galactic Dynamics*; Princeton University Press: Princeton, NJ, USA, 1987.

The Virial Theorem – Newtonian Version

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Binney, J.; & Tremaine, S. *Galactic Dynamics*; Princeton University Press: Princeton, NJ, USA, 1987.

Zwicky's Estimate

$$M = \frac{r_g \langle v^2 \rangle}{G}$$

→ **Dark Matter**

The Virial Theorem and Retarded Gravity

The Correct Virial Theorem

$$\frac{d\bar{G}}{dt} = 2T + \sum_{k=1}^{N} \vec{F}_{Nk} \cdot \vec{r}_k + \sum_{k=1}^{N} \vec{F}_{rk} \cdot \vec{r}_k,$$

$$V_{rT} = \sum_{k=1}^{N} \vec{F}_{rk} \cdot \vec{r}_k = -\sum_{k=1}^{N} m_k \vec{\nabla}_{\vec{r}_k} \phi_r \cdot \vec{r}_k,$$

The Correct Virial Theorem

$$\rho(\vec{x}) = \sum_{k=1}^{N} m_k \delta^{(3)}(\vec{x} - \vec{r}_k) \Rightarrow$$

$$V_{rT} = \int \rho(\vec{x}) \vec{\nabla} \phi_r \cdot \vec{x} d^3x = -\frac{G}{2c^2} \int \rho(\vec{x}) \vec{\nabla} \left(\int R \rho^{(2)}(\vec{x}') d^3x' \right) \cdot \vec{x} d^3x$$

$$= -\frac{G}{2c^2} \int \int d^3x d^3x' \rho(\vec{x}) \frac{\vec{R} \cdot \vec{x}}{R} \rho^{(2)}(\vec{x}') = -G \int \int d^3x d^3x' \frac{\rho(\vec{x}) \rho_d(\vec{x}')}{R}.$$

In which we defined "dark matter density" as follows:

$$\rho_d \equiv \rho^{(2)} \frac{\vec{R} \cdot \vec{x}}{2c^2}$$

The Correct Virial Theorem

$$2\langle T\rangle = \langle |V_{NT}|\rangle - \langle V_{rT}\rangle ,$$

The Correct Virial Theorem

$$M_d \equiv \frac{|V_{rT}| r_g}{GM} \Rightarrow \left\langle v^2 \right\rangle_t = \frac{G(M+M_d)}{r_g} \Rightarrow M_d = \frac{r_g \left\langle v^2 \right\rangle_t}{G} - M.$$

This result of course does not allude to some mysterious "dark matter" but rather tells us something on the dynamics of the galaxy cluster itself.

The "Uncertainty Relation" of Retarded Gravity

We already know that the gravitational force generated by particle j on particle k is:

$$\begin{aligned}
\vec{F}_{j,k} &= \vec{F}_{Nj,k} + \vec{F}_{rj,k} \\
\vec{F}_{Nj,k} &= -G\frac{m_j m_k}{R_{k,j}^2}\hat{R}_{k,j}, \qquad \vec{R}_{k,j} \equiv \vec{r}_k - \vec{r}_j,\ R_{k,j} \equiv |\vec{R}_{k,j}(t)|,\ \hat{R}_{k,j} \equiv \frac{\vec{R}_{k,j}}{R_{k,j}}, \\
\vec{F}_{rj,k} &= \frac{G m_j m_k}{2R_{k,j}^2 c^2}\left(R_{k,j}\vec{a}_{\perp j,k} + \hat{R}_{k,j}\vec{v}_{\perp j,k}^2 - 2(\vec{v}_{j,k}\cdot\hat{R}_{k,j})\vec{v}_{\perp j,k}\right) \\
\vec{a}_{\perp j,k} &\equiv \vec{a}_j - (\vec{a}_j \cdot \hat{R}_{k,j})\hat{R}_{k,j}, \qquad \vec{v}_{\perp j,k} \equiv \vec{v}_j - (\vec{v}_j \cdot \hat{R}_{k,j})\hat{R}_{k,j}.
\end{aligned}$$

But which one is more important?

We already answered that this depends on the distance, for very large distances the retardation force become more important bdespite the $1/c^2$ term since it drops only as $1/r$ (and not as $1/r^2$). Let us put this in more precise terms for a slow point particle. Let's look at the acceleration of a test particle in the vicinity of massive particle j.

$$a_{Nj} = |\vec{a}_{Nj}| = G\frac{m_j}{R_j^2} \quad \Rightarrow \quad \vec{a}_{rj} = a_{Nj}\frac{\left(R_j \vec{a}_{\perp j} + \hat{R}_j \vec{v}^2_{\perp j} - 2(\vec{v}_j \cdot \hat{R}_j)\vec{v}_{\perp j}\right)}{2c^2}$$

$$\beta \equiv \frac{v}{c} \ll 1,$$

→

$$\vec{a}_{rj} \simeq a_{Nj} \frac{R_j \vec{a}_{\perp j}}{2c^2}, \qquad \vec{a}_{Tj} = \vec{a}_{Nj} + \vec{a}_{rj} \simeq a_{Nj} \left(-\hat{R}_j + \frac{R_j \vec{a}_{\perp j}}{2c^2} \right).$$

"Uncertainty" Relation

$$\frac{R_j a_{\perp j}}{2c^2} > 1 \quad \Rightarrow \quad R_j a_{\perp j} > 2c^2 \quad \Rightarrow \quad a_{\perp j} > a_c = \frac{2c^2}{R_j}.$$

Consider a point mass located on a circle which serves as border of the M33 galaxy, then we may ask what will be the amount of acceleration suffered by the point mass that will cause a retardation effect on a test particle located across the diameter of the galaxy (which is the furthest point on the imaginary circle from the point mass). Now the radius of the galaxy M33 is $Rs \simeq 30,000$ light years $= 2.8\ 10^{20}$ meters. Hence we will need an acceleration of about:

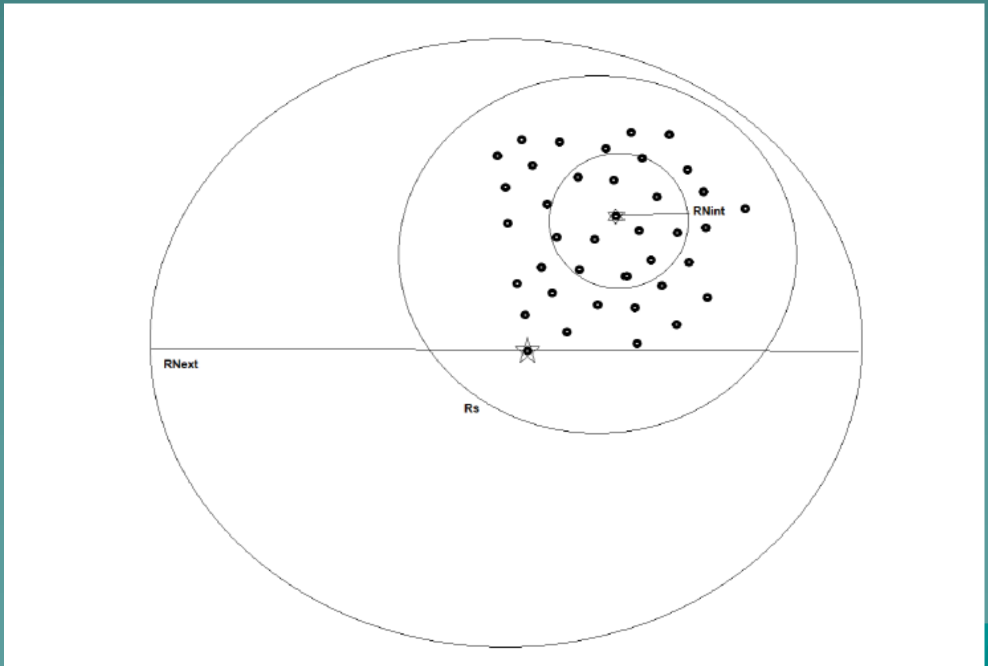
$$a_c = \frac{2c^2}{R_j} = \frac{2c^2}{2Rs} = \frac{c^2}{Rs} \simeq 0.00032 \text{ m/s}^2$$

to observe the effect of retarded gravity. This does not seem to be such a huge acceleration and many point masses (atoms, molecules etc.) in the galaxy may have accelerations that need to be considered in the total galactic balance of gravitational forces.

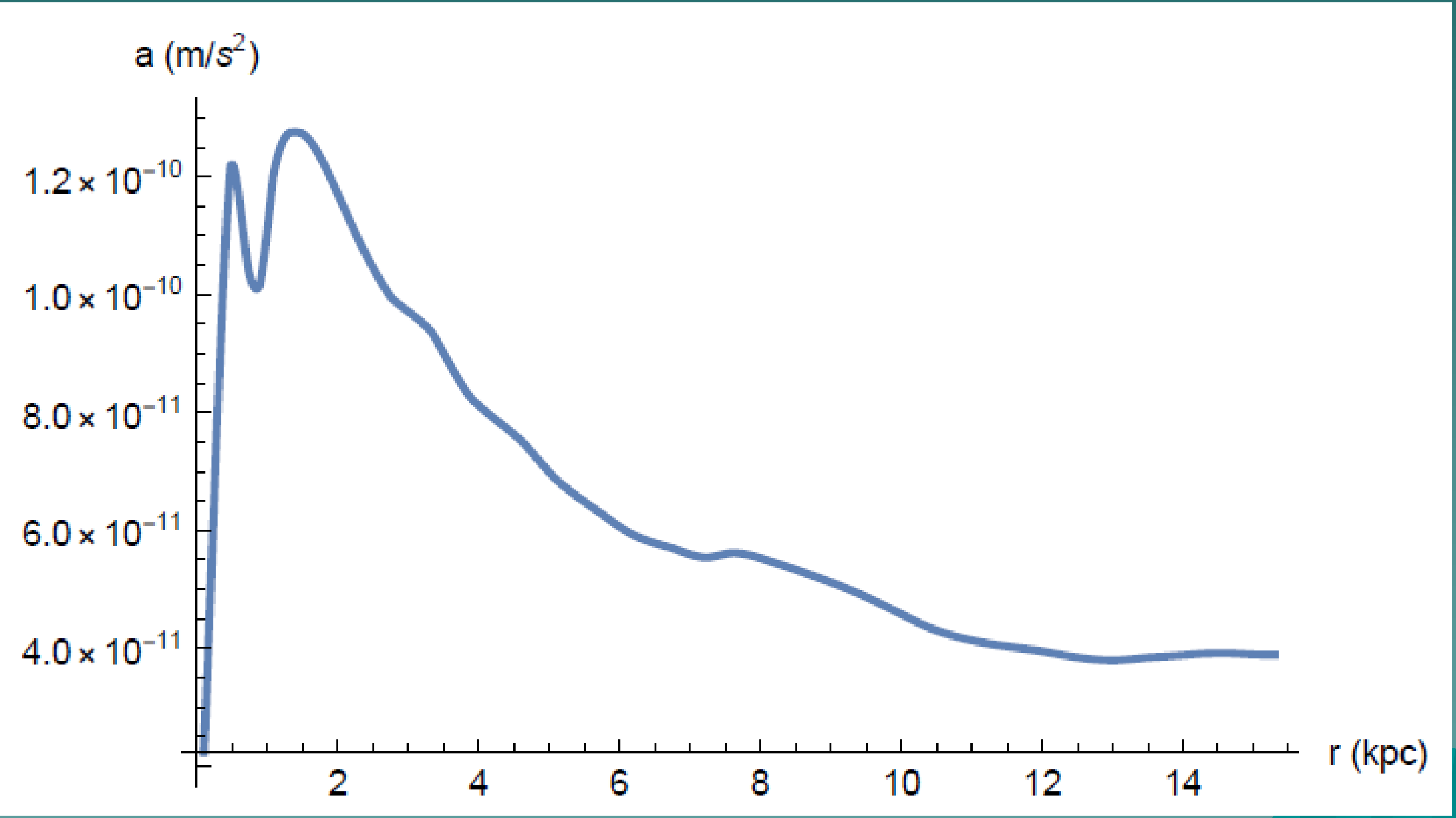
The acceleration of the Earth in its orbit around the Sun is approximately $5.95 \cdot 10^{-3}$ m/s^2.

The Definition of the Newtonian regime:

$$\frac{R_j a_{\perp j}}{2c^2} < 1 \qquad R_j < R_{Nj} \equiv \frac{2c^2}{a_{\perp j}}.$$



Rotational acceleration in M33:



Newtonian radius is much bigger than the galaxy!

Defining “Dark Matter”

$$\phi = -G\int \frac{\rho(\vec{x}', t)}{R} d^3x' + \frac{G}{c}\int \rho^{(1)}(\vec{x}', t) d^3x' - \frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t) d^3x'$$

$$\phi_r = -\frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t) d^3x'$$

$$\begin{aligned}
\vec{F} &= \vec{F}_N + \vec{F}_r \\
\vec{F}_N &= -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}', t)}{R^2}\hat{R} d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R} \\
\vec{F}_r &\equiv -\vec{\nabla}\phi_r = \frac{G}{2c^2}\int \rho^{(2)}(\vec{x}', t)\hat{R} d^3x'
\end{aligned}$$

For large distances:

$$\hat{R} \simeq \frac{\vec{x}}{|\vec{x}|} \equiv \hat{r}$$

The rough retardation approximation:

$$\vec{F}_r = \frac{G}{2c^2}\hat{r}\int \rho^{(2)}(\vec{x}', t) d^3x' = \frac{G}{2c^2}\hat{r}\ddot{M}, \qquad \ddot{M} \equiv \frac{d^2 M}{dt^2}.$$

Retardation forces can be repulsive or attractive. We notice that this approximation is not always appropriate, but allows us to encapsulate retardation using a single parameter which is useful.

$$\dot{M} > 0$$

Mass is accreted from the intergalactic gas.

$$\ddot{M} < 0$$

The intergalactic gas is depleted.

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r}$$

The retardation force is attractive in the galactic scenario.

Defining “dark matter”

$$-\frac{v_c^2}{r}\hat{r} = \vec{F}_d = -\frac{G M_d(r)}{r^2}\hat{r}$$

However, F_d is really F_r

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r}$$

→

$$M_d(r) = \frac{r^2|\ddot{M}|}{2c^2}$$

$$M_d(r) = 4\pi \int_0^r r'^2 \rho_d(r') dr', \qquad \frac{dM_d(r)}{dr} = 4\pi r^2 \rho_d(r)$$

$$\rho_d(r) = \frac{|\ddot{M}|}{4\pi c^2 r}$$

"dark matter" density decreases as $r^{-1.3}$ for the M33 galaxy.

E. Corbelli; P. Salucci (2000). "The extended rotation curve and the dark matter halo of M33". Monthly Notices of the Royal Astronomical Society. 311 (2): 441447. arXiv:astro-ph/9909252. doi:10.1046/j.1365-8711.2000.03075.x.

Mass Conservation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial^2 \rho}{\partial t^2} + \vec{\nabla} \cdot \left(\frac{\partial \rho}{\partial t} \vec{v} + \rho \frac{\partial \vec{v}}{\partial t}\right) = 0.$$

$$\frac{1}{c^2}\ddot{M} = \frac{1}{c^2}\int \frac{\partial^2 \rho}{\partial t^2} d^3x = \oint d\vec{S} \cdot \left(\frac{\vec{v}}{c}\left(\frac{\vec{v}}{c} \cdot \vec{\nabla}\rho + \rho \vec{\nabla} \cdot \left(\frac{\vec{v}}{c}\right)\right) - \rho \frac{1}{c}\frac{\partial \frac{\vec{v}}{c}}{\partial t}\right).$$

“Dark Mass”

$$M_d(r) = \frac{r^2}{2}\oint d\vec{S}\cdot\left(\frac{\vec{v}}{c}\left(\frac{\vec{v}}{c}\cdot\vec{\nabla}\rho+\rho\vec{\nabla}\cdot(\frac{\vec{v}}{c})\right)-\rho\frac{1}{c}\frac{\partial\frac{\vec{v}}{c}}{\partial t}\right).$$

$$\left[\frac{M_d(r)}{M}\right]\approx\left(\frac{v}{c}\right)^2\left[\frac{r}{l_\rho}+\frac{r}{l_v}+\frac{r}{l_d}\right]$$

$$l_\rho\equiv\frac{\rho}{|\vec{\nabla}\rho|},\qquad l_v\equiv\frac{v}{|\vec{\nabla}\cdot v|},\qquad l_d\equiv\frac{v^2}{|\partial_t v|}$$

Retardation vs. Newtonian Forces

$$\frac{1}{l_t} = \left[\frac{1}{l_\rho} + \frac{1}{l_v} + \frac{1}{l_d}\right]$$

$$\frac{F_r}{F_N} = \left[\frac{M_d(r)}{M}\right] \approx \left(\frac{v}{c}\right)^2 \left[\frac{r}{l_t}\right]$$

$$\left(\frac{v}{c}\right)^2 \approx 10^{-6}$$

$$\frac{r}{l} \approx 10^6$$

Acceleration Condition

MOND corrections are needed for small acceleration, so let us write the conditions needed for retardation corrections in the language of acceleration.

$$a = \frac{v^2}{r}$$

$$fr \equiv \frac{F_r}{F_N}$$

⇒

$$\frac{v^2}{c^2}\frac{r}{l_t} > fr$$

⇒

$$a = \frac{v^2}{r} > a_c(r) \equiv fr\frac{c^2 l_t}{r^2}$$

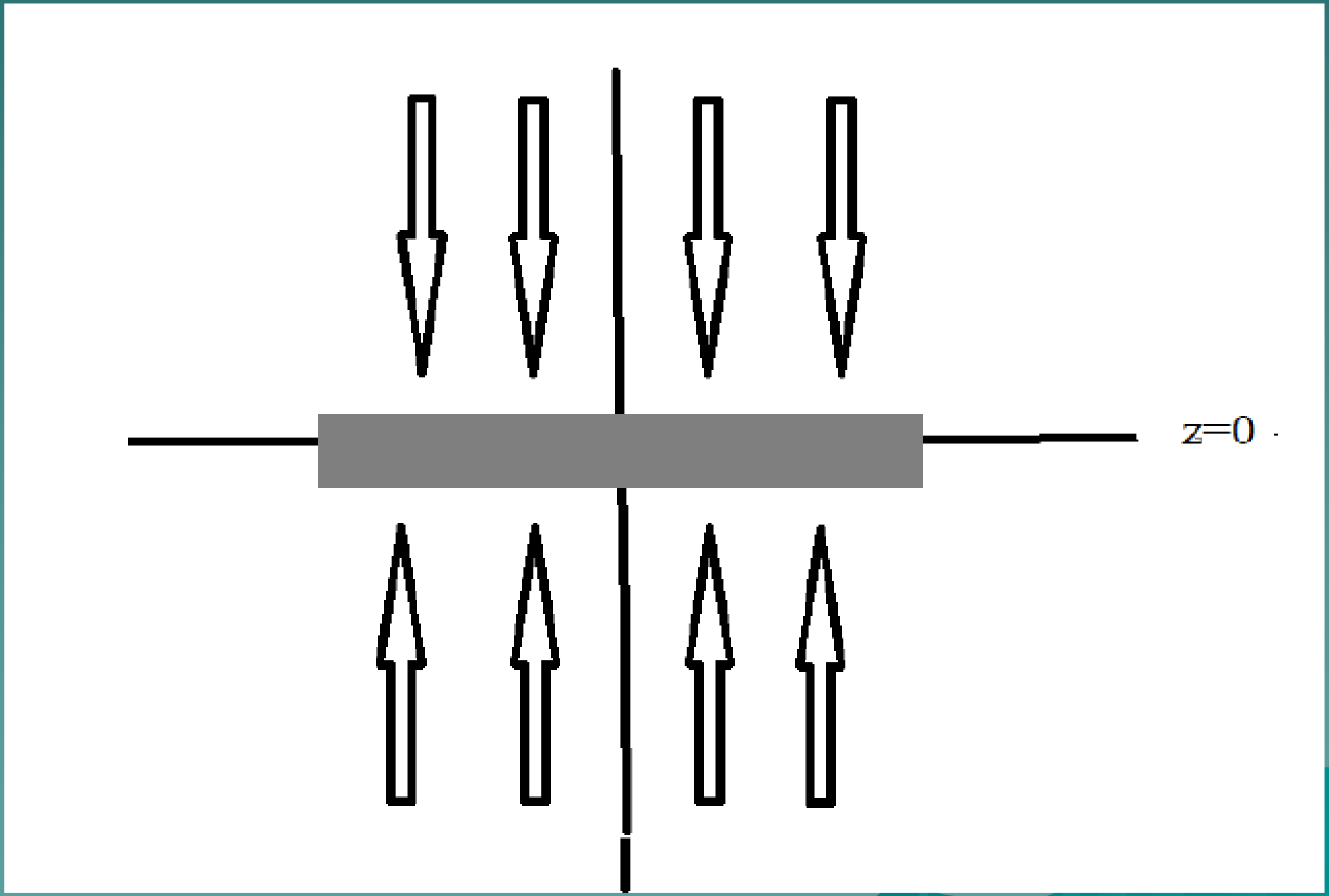
Acceleration Condition

$$a = \frac{v^2}{r} > a_c(r) \equiv f r \frac{c^2 l_t}{r^2}$$

However, as $a_c(r)$ decreases as r^{-2} this means that this inequality will be satisfied more easily for larger r. Thus "dark matter" effects can be interpreted in terms of acceleration as MOND postulates.

However, acceleration does not need to be small or equal with respect to a_0 in order to have "dark matter" effects, rather acceleration must be bigger than some critical acceleration $a_c(r)$ which depends on radial distance. Thus, the inequality becomes easier to satisfy at large radial distances, in which case the acceleration <u>is indeed quite small as MOND suggests</u>.

Depletion



$$(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p(\rho)}{\rho} - \vec{\nabla}\phi$$

$$\vec{v}\cdot\vec{\nabla} = v_z\frac{\partial}{\partial z} + \frac{v_\theta}{\bar{r}}\frac{\partial}{\partial\theta}$$

$$\Rightarrow$$

$$\frac{v_\theta^2}{\bar{r}} \simeq \frac{\partial\phi}{\partial\bar{r}},$$

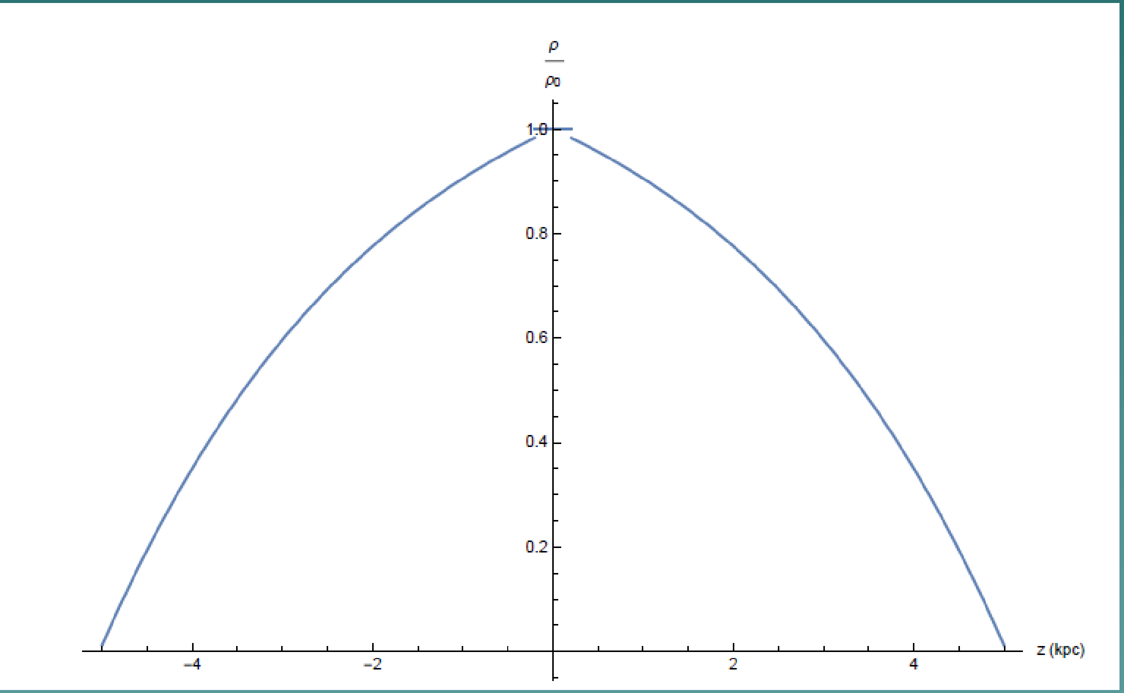
$$\frac{1}{2}v_z^2 + w(\rho) + \phi = C(r,t).$$

$$w(\rho) = \int\frac{dP}{\rho}$$

Dynamical Equilibrium

$$v_z = \begin{cases} -|v_z| & z > 0 \\ |v_z| & z < 0 \end{cases}$$

Depletion



$$\rho_o(\bar{r}, z, 0) = re(z)\left[\rho_1(\bar{r}) + \rho_2(\bar{r})e^{k|z|}\right],$$

$$re(z) = \begin{cases} 1 & |z| < z_i \\ 0 & |z| \geq z_i \end{cases}$$

Density distribution outside the galaxy

$$\rho_o(\bar{r}, z, t) = \frac{\gamma}{v_z} = re(z - v_z t)[\rho_1(\bar{r}) + \rho_2(\bar{r}) e^{k|z - v_z t|}]$$

Mass outside the galaxy

$$M_o(t) = 2\pi\left[\int_{-z_i}^{-\frac{1}{2}\Delta z} dz \int_0^{r_m} d\bar{r}\bar{r}\rho_o(\bar{r},z,t) + \int_{\frac{1}{2}\Delta z}^{z_i} dz \int_0^{r_m} d\bar{r}\bar{r}\rho_o(\bar{r},z,t)\right]$$

Mass inside the galaxy

$$M(t) = M_T - M_o(t)$$

$$\dot{M}(t) = -\dot{M}_o(t), \qquad \ddot{M}(t) = -\ddot{M}_o(t)$$

Mass inside the galaxy

$$\ddot{M}(t) = \ddot{M}(0)e^{\frac{t}{\tau}}$$

$$\alpha \equiv k|v_z|, \qquad \tau \equiv \frac{1}{\alpha}$$

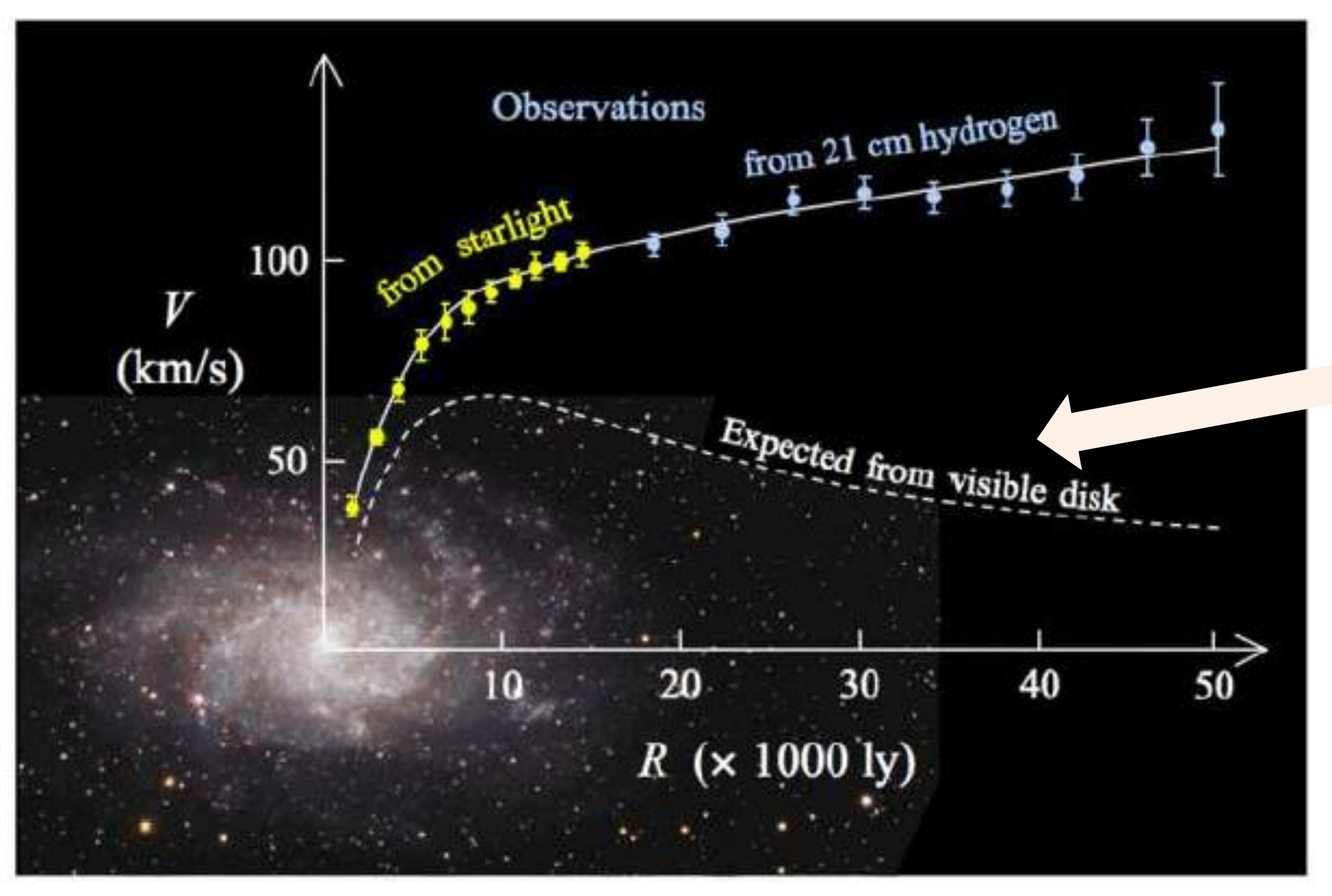
Mass inside the galaxy

$$\dot{M}(t) = \dot{M}(0) + \tau\ddot{M}(0)\left(e^{\frac{t}{\tau}} - 1\right) = \dot{M}(0) - \tau|\ddot{M}(0)|\left(e^{\frac{t}{\tau}} - 1\right).$$

$$M(t) = M(0) + (\dot{M}(0) - \tau\ddot{M}(0))\,t + \tau^2\ddot{M}(0)\left(e^{\frac{t}{\tau}} - 1\right), \quad \tau > 0.$$

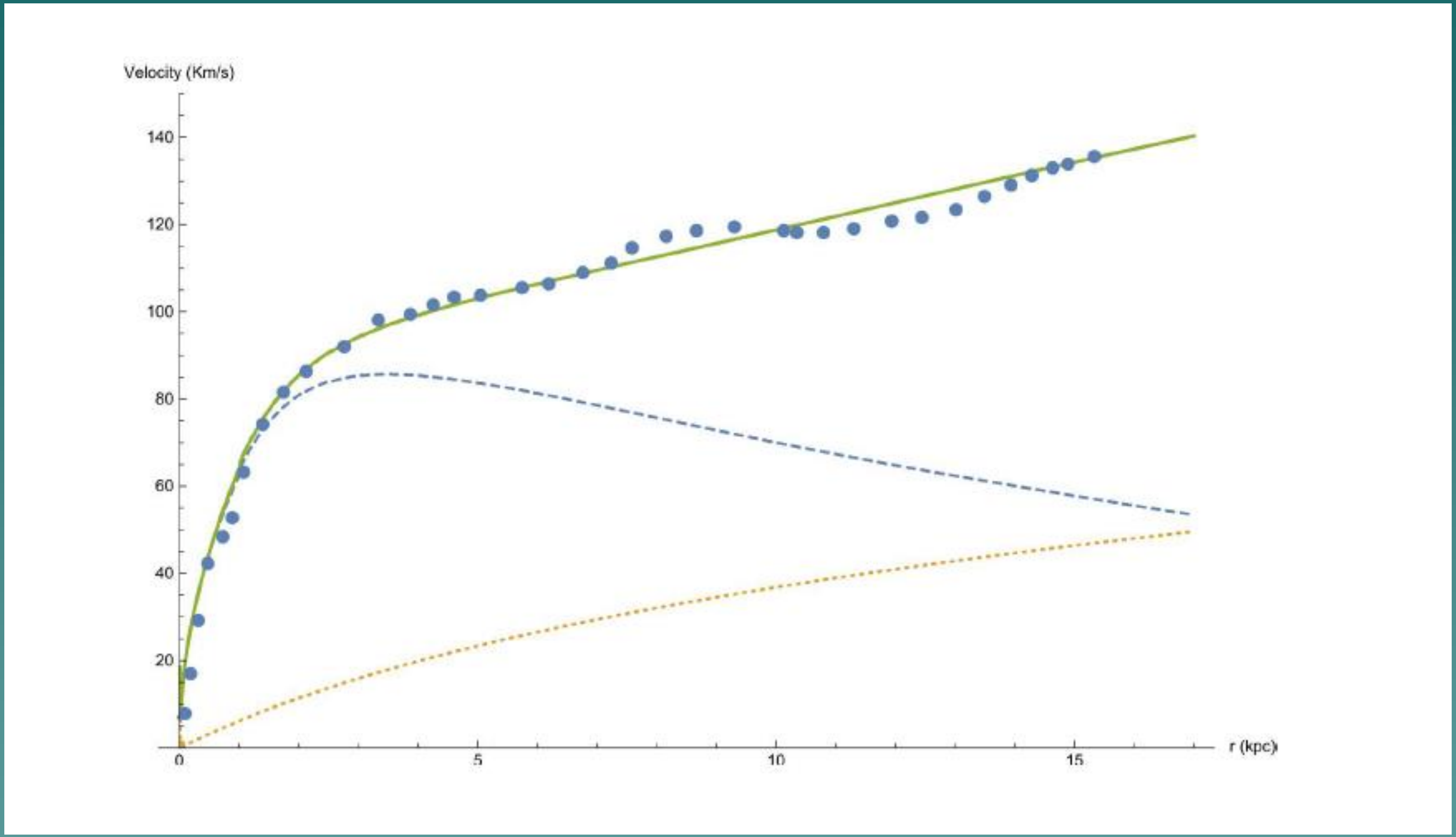
Some Results for Various Galaxies

Strange Rotation Curves

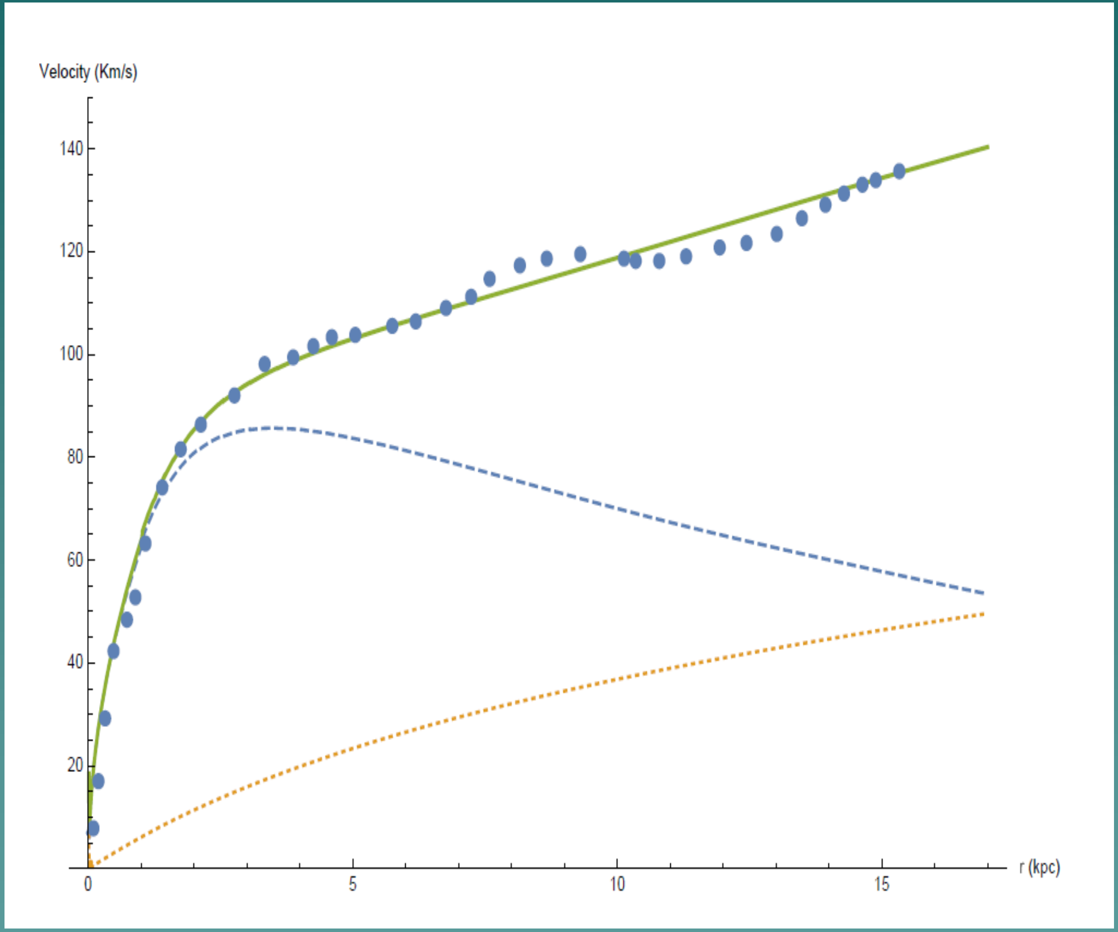


If you forget about retardation

M33



Rotation curve for M33. The observational points were supplied by Dr. Michal Wagman, a former PhD student at Ariel University, under my supervision. The full line describes the complete rotation curve, which is the sum of the dotted line, describing the retardation contribution, and the dashed line, which is the Newtonian contribution.



Detailed modelling: M/L =1

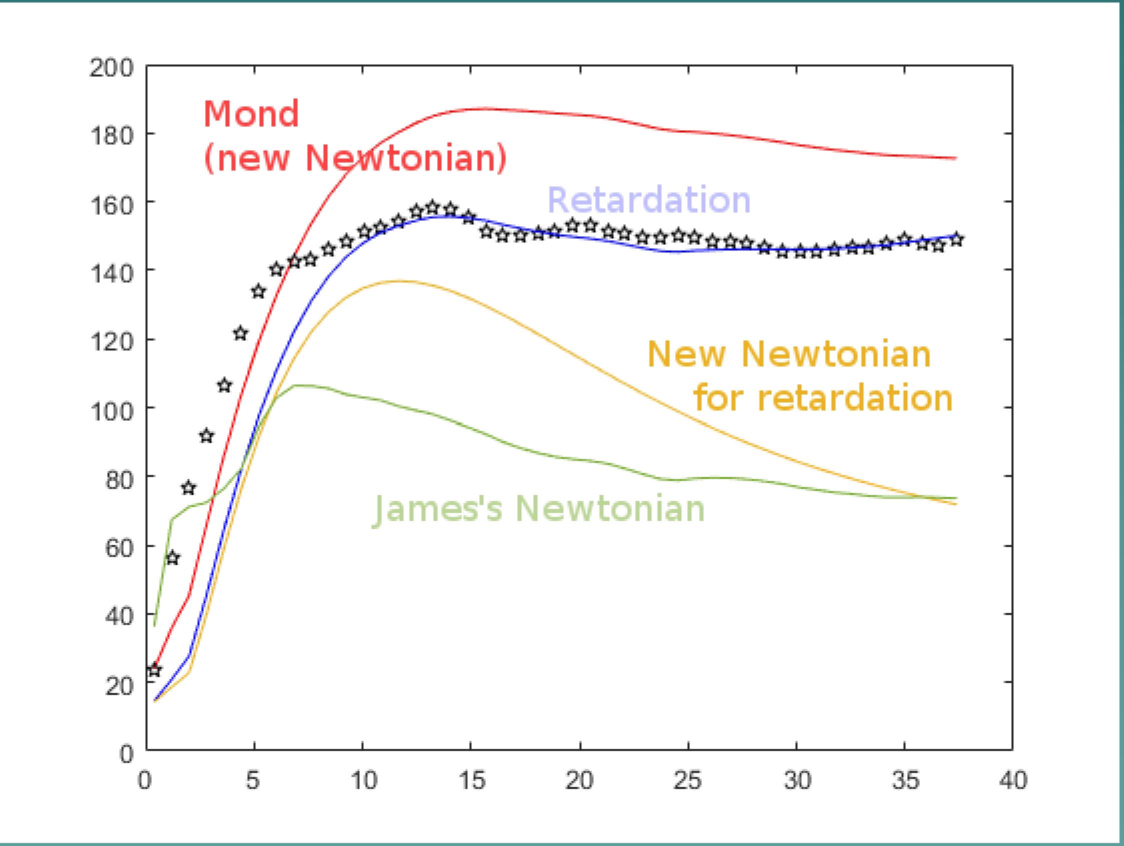
M/L is not a fitting parameter as most authors assume.

$$|\ddot{M}| = \frac{M}{t_r^2} \simeq 9.12 \times 10^{16}\ \mathrm{kg/s^2}$$

the full line describe the complete rotation curve which is the sum of the dotted line describing the retardation contribution and the dashed line which is the Newtonian contribution.

Work by Michal Wagman former PhD student at Ariel University.

NGC 3198



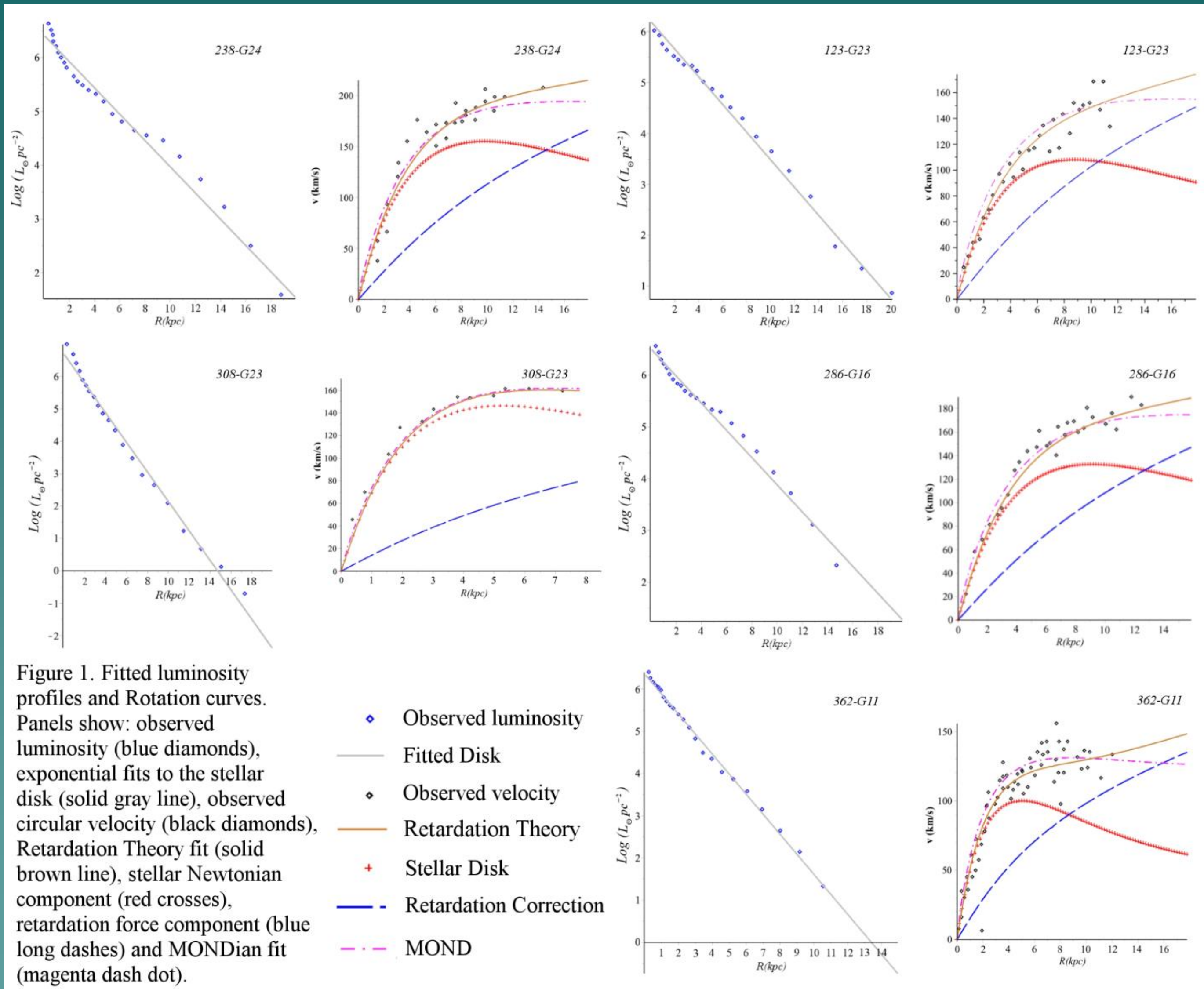


Figure 1. Fitted luminosity profiles and Rotation curves. Panels show: observed luminosity (blue diamonds), exponential fits to the stellar disk (solid gray line), observed circular velocity (black diamonds), Retardation Theory fit (solid brown line), stellar Newtonian component (red crosses), retardation force component (blue long dashes) and MONDian fit (magenta dash dot).

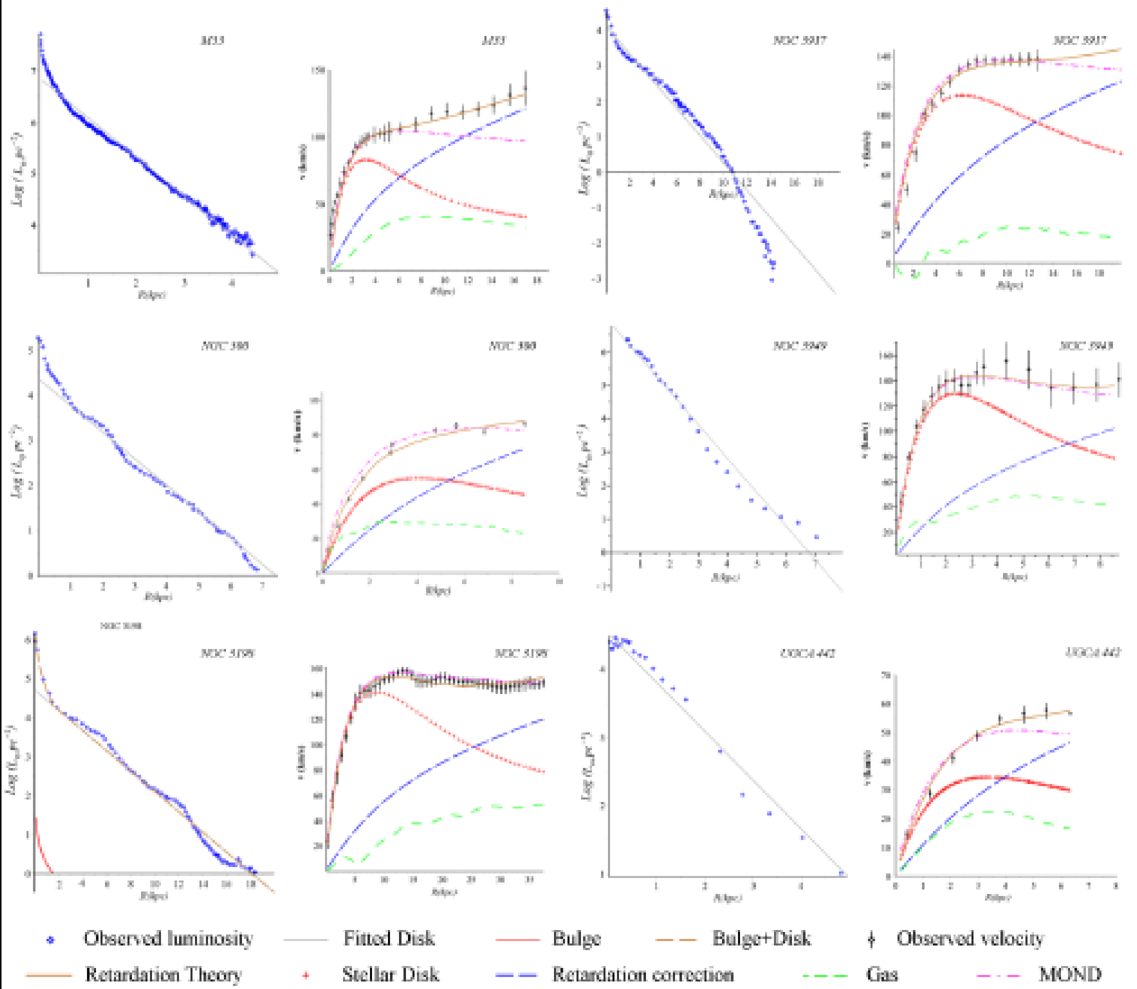
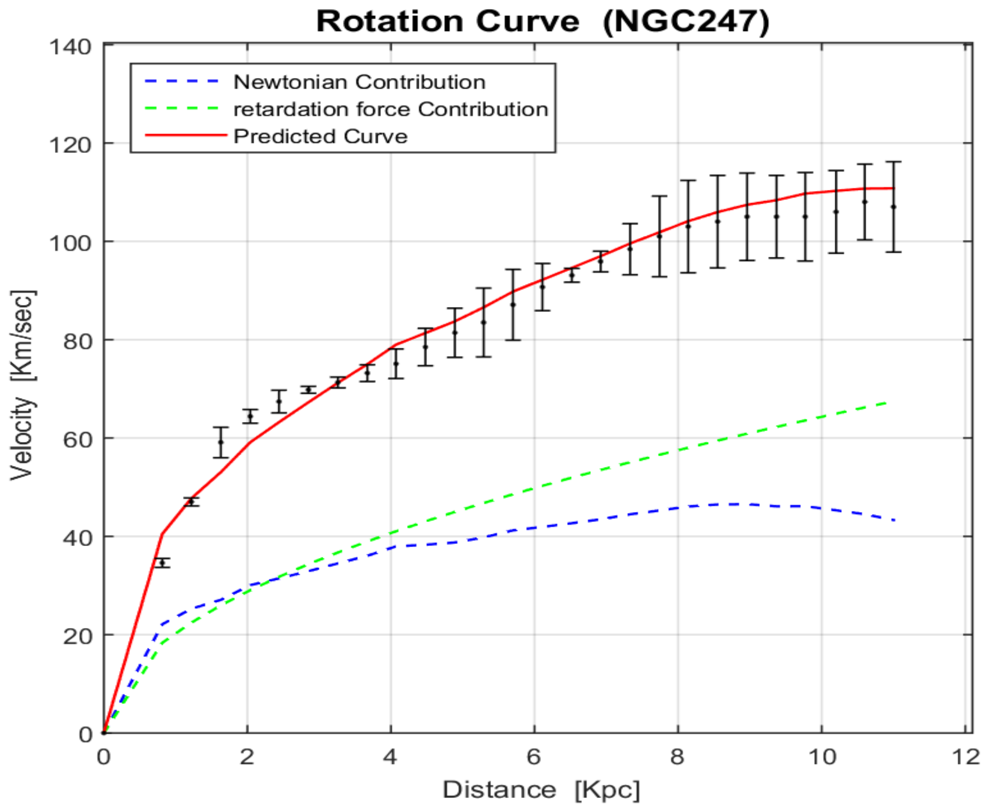


Figure 2. Fitted luminosity profiles and Rotation curves. Panels show: observed luminosity (blue diamonds), exponential fits to the stellardisk (solid gray line), bulge fit (red solid line), sum of bulge+disk (long brown dashes), observed circular velocity (black diamonds w/ error bar), Retardation Theory fit (solid brown line), stellar Newtonian component (red crosses), retardation force component (blue long dashes), gas component (short green dashes) and MONDian fit (magenta dash dot).

Work by Tomer Zimmerman & Roy Gomel
PhD students at Tel Aviv University



Mdotim = 3.61e+16 [kg

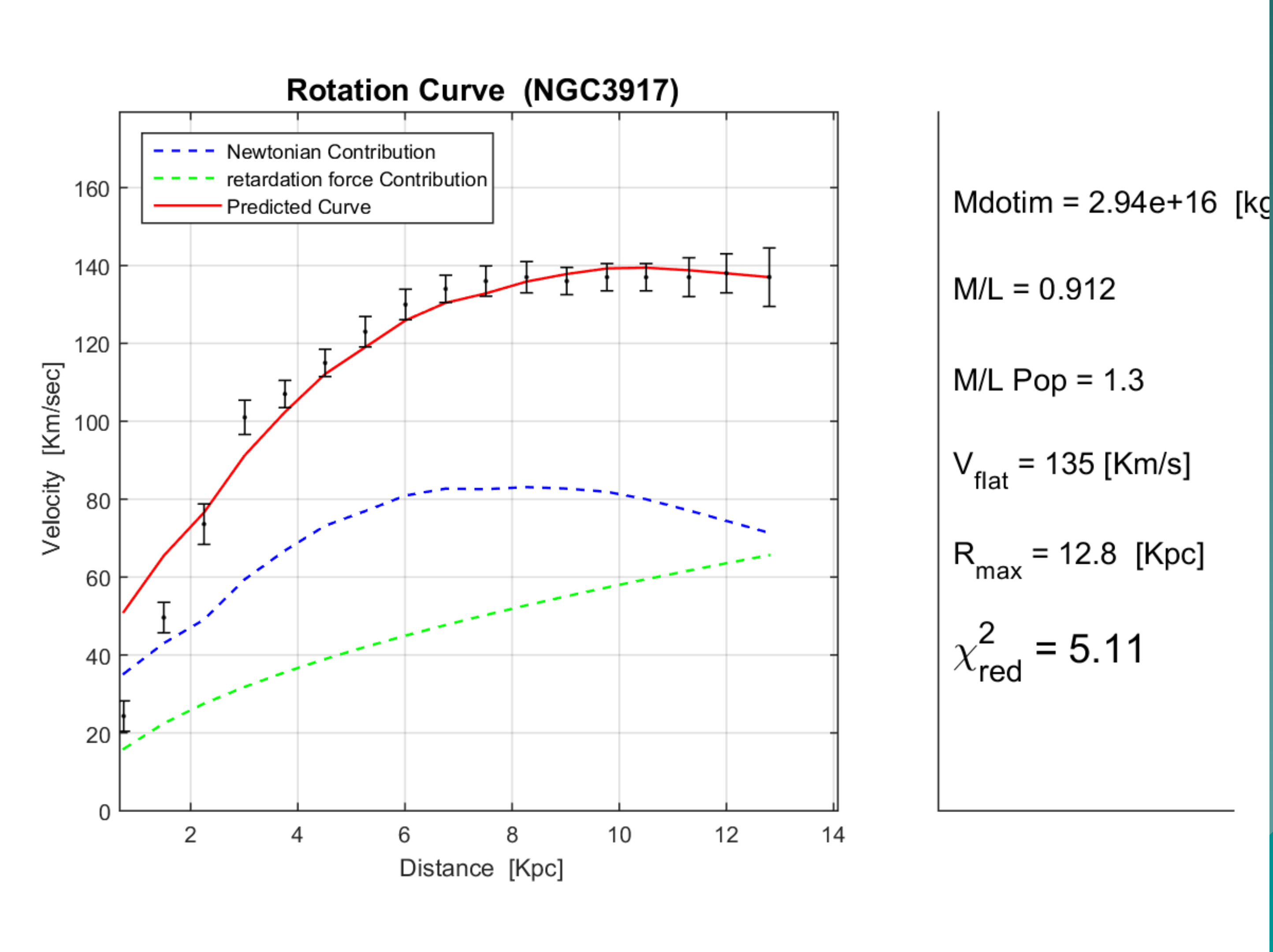
M/L = 0.628

M/L Pop = 1

V_{flat} = 107 [Km/s]

R_{max} = 11 [Kpc]

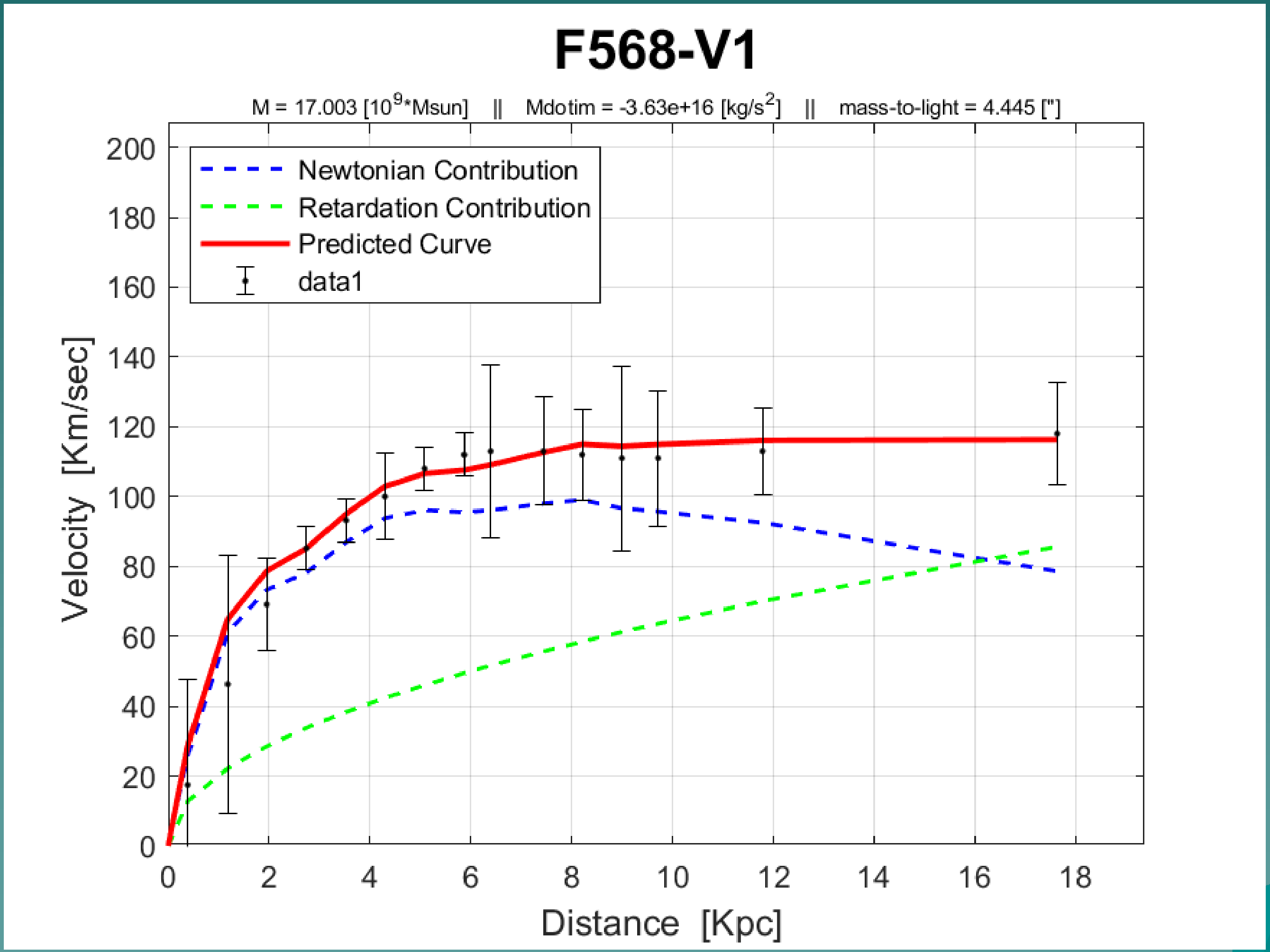
$\chi^2_{red} = 3.38$

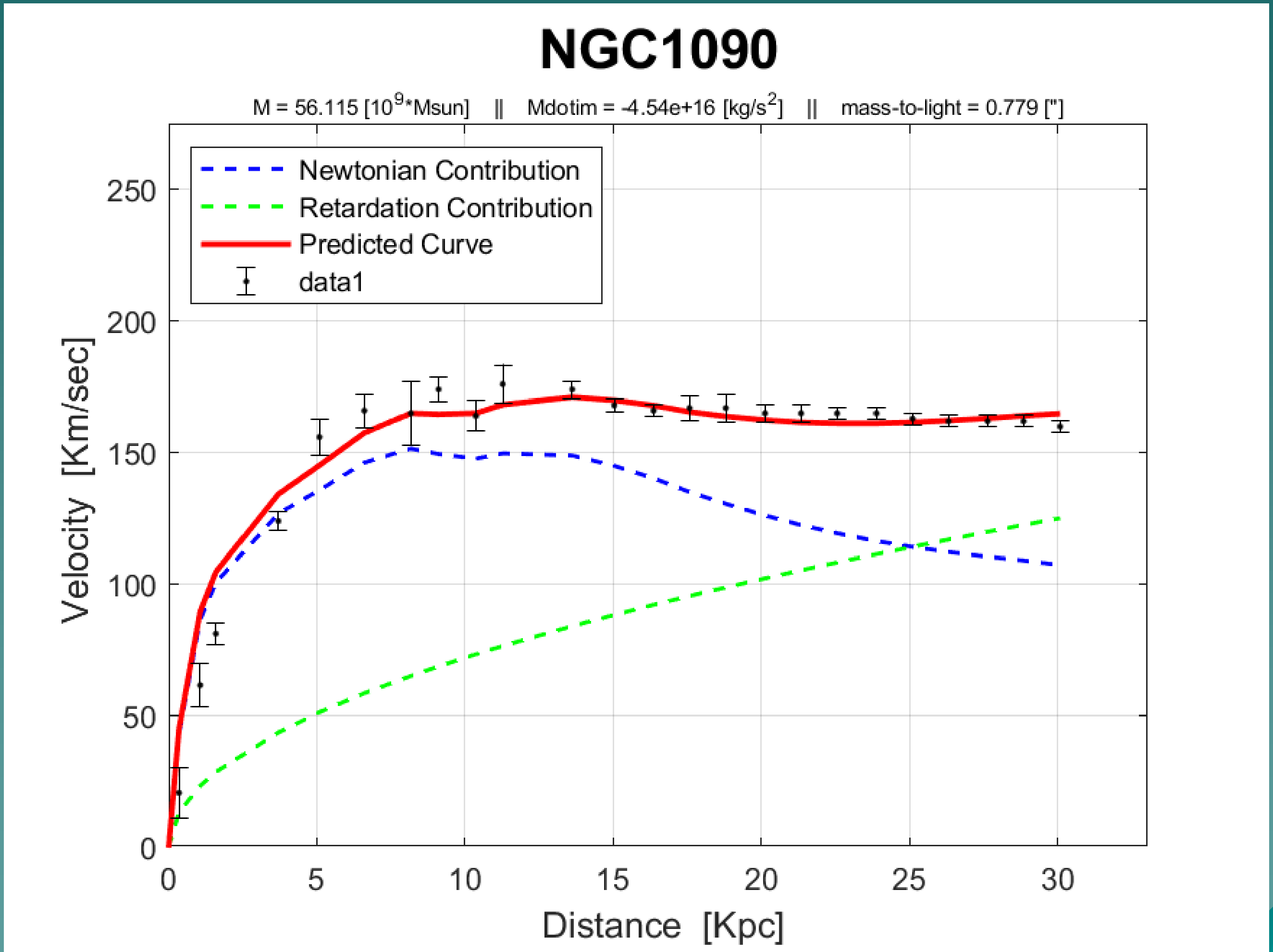

Rotation Curve (NGC3917)
Newtonian Contribution
retardation force Contribution
Predicted Curve
Velocity [Km/sec]
Distance [Kpc]
0
20
40
60
80
100
120
140
160
2
4
6
8
10
12
14
Mdotim = 2.94e+16 [kg
M/L = 0.912
M/L Pop = 1.3
V_{flat} = 135 [Km/s]
R_{max} = 12.8 [Kpc]
χ^2_{red} = 5.11

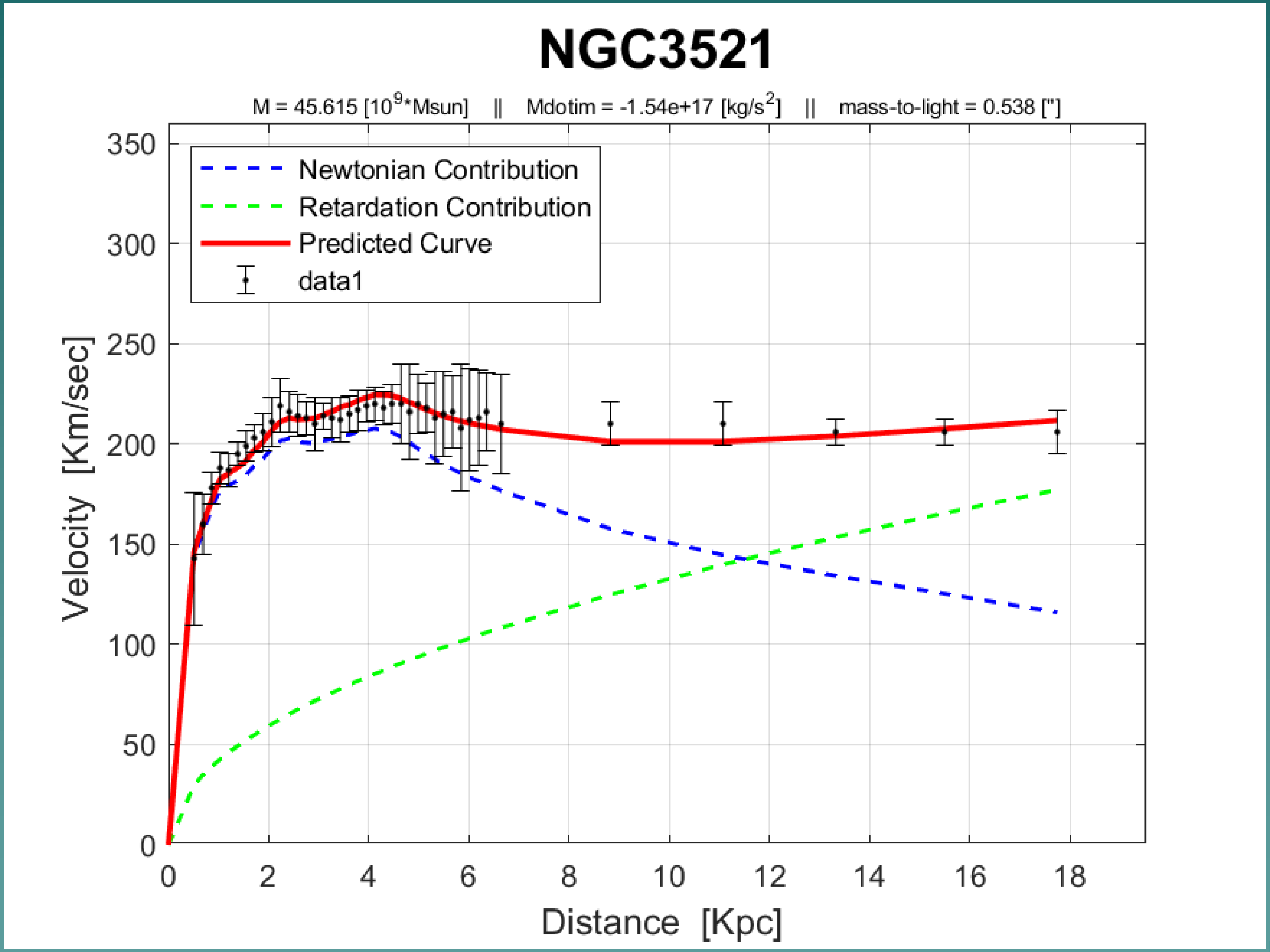
Yuval Glass & Tomer Zimmerman

1. Rough approximation to retarded gravity
2. Automation of the fitting process
3. 143 Galaxies in 20 minutes.

Glass, Yuval, Tomer Zimmerman, and Asher Yahalom. 2024. "Retarded Gravity in Disk Galaxies" Symmetry 16, no. 4: 387. https://doi.org/10.3390/sym16040387


F568-V1
M = 17.003 [10^9*Msun] || Mdotim = -3.63e+16 [kg/s^2] || mass-to-light = 4.445 ["]
Newtonian Contribution
Retardation Contribution
Predicted Curve
data1
Velocity [Km/sec]
Distance [Kpc]
0
20
40
60
80
100
120
140
160
180
200
0
2
4
6
8
10
12
14
16
18


NGC1090
M = 56.115 [10^9*Msun] || Mdotim = -4.54e+16 [kg/s^2] || mass-to-light = 0.779 ["]
Newtonian Contribution
Retardation Contribution
Predicted Curve
data1
Velocity [Km/sec]
0
50
100
150
200
250
0
5
10
15
20
25
30
Distance [Kpc]


NGC3521
M = 45.615 [10^9*Msun] || Mdotim = -1.54e+17 [kg/s^2] || mass-to-light = 0.538 ["]
Newtonian Contribution
Retardation Contribution
Predicted Curve
data1
Velocity [Km/sec]
Distance [Kpc]
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

How not to interpret retardation theory

$$|\ddot{M}| = \frac{M}{t_r^2} \simeq 9.12 \times 10^{16}\ \mathrm{kg/s^2}$$

Is not constant over large time scales!

$$\ddot{M}(t) = \ddot{M}(0) e^{\frac{t}{\tau}}$$

How not to interpret retardation theory

If it is assumed constant:

$$\dot{M}(t) = \dot{M}(0) + t\ddot{M} = \dot{M}(0) - t|\ddot{M}|$$

$$\dot{M}(0) = T|\ddot{M}|$$

How not to interpret retardation theory

If it is assumed constant:

$$M(t) = M(0) + \dot{M}(0)t + \frac{1}{2}\ddot{M}t^2$$

$$M(t) = \ddot{M}t(\frac{1}{2}t - T) \Rightarrow M(T) = -\frac{1}{2}\ddot{M}T^2 = \frac{1}{2}|\ddot{M}|T^2$$

How not to interpret retardation theory

By plugging in the mass accumulation decrease rate, we arrive at $M(T) \simeq 7.66 \times 10^{51}$ kg, which is clearly 11 orders of magnitude greater than the known mass of the galaxy.

However, the second derivative of M is clearly not constant as dictated by the dynamics.

$$\ddot{M}(t) = \ddot{M}(0) e^{\frac{t}{\tau}}$$

Higher order effects

Are retardation effects result of a Taylor series approximation? No.

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}$$

$$\begin{aligned} \phi &= \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' \\ &= -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x' \end{aligned}$$

The force is always partitioned to a retarded Newtonian force and a retardation force that has no parallel in Newtonian theory.

$$\vec{F} = \vec{F}_{Nr} + \vec{F}_r$$

$$\vec{F}_{Nr} = -G \int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R^2} \hat{R} d^3 x', \qquad \hat{R} \equiv \frac{\vec{R}}{R}$$

$$\vec{F}_r \equiv -\frac{G}{c} \int \frac{\rho^{(1)}(\vec{x}', t - \frac{R}{c})}{R} \hat{R} d^3 x', \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}.$$

Conservation of Mass

Question:

How do we know that the correction of the gravitational potential created by the negative second derivative of the mass of the galaxy is not cancelled out by the gravitational effect of the necessarily positive second derivative of the mass of the IGM.

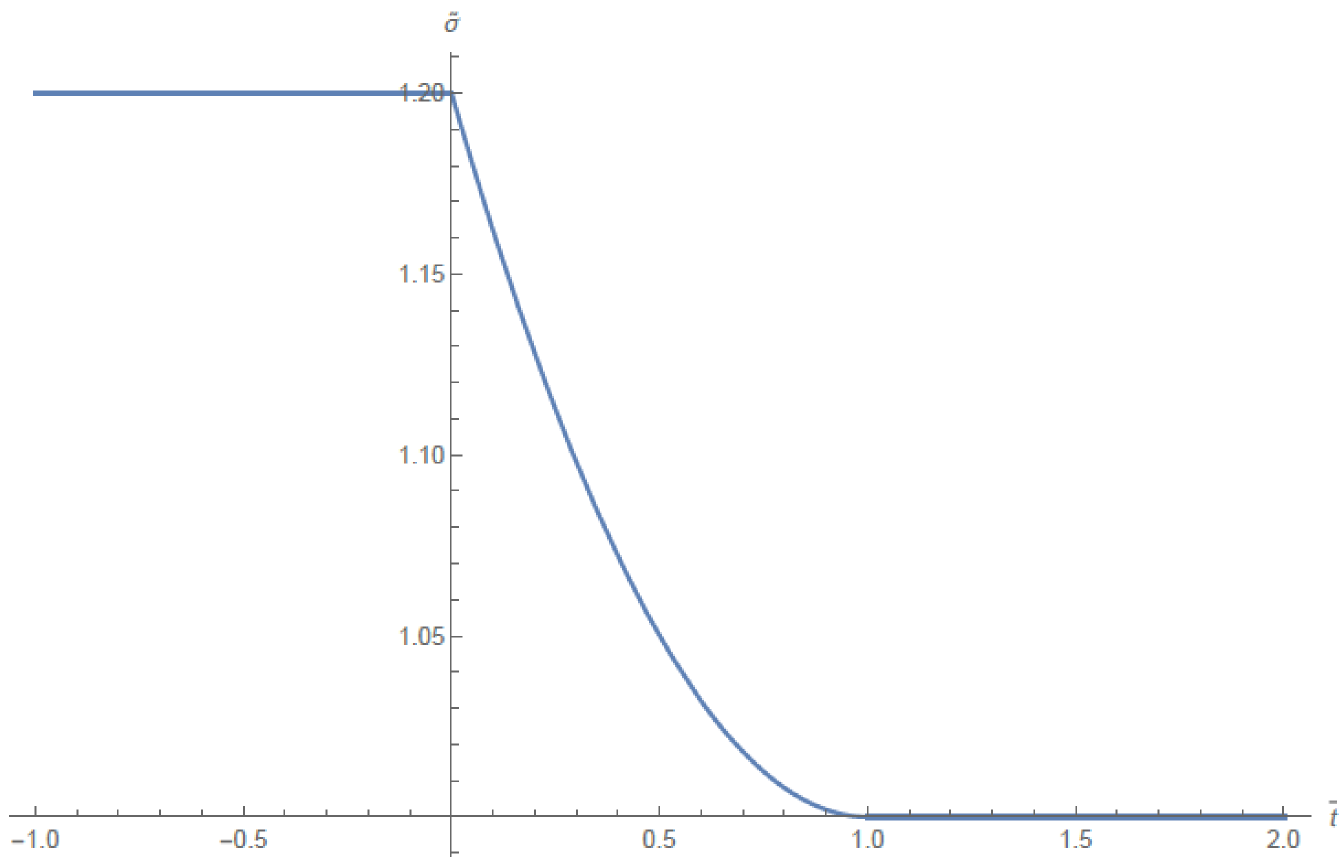
Answer:

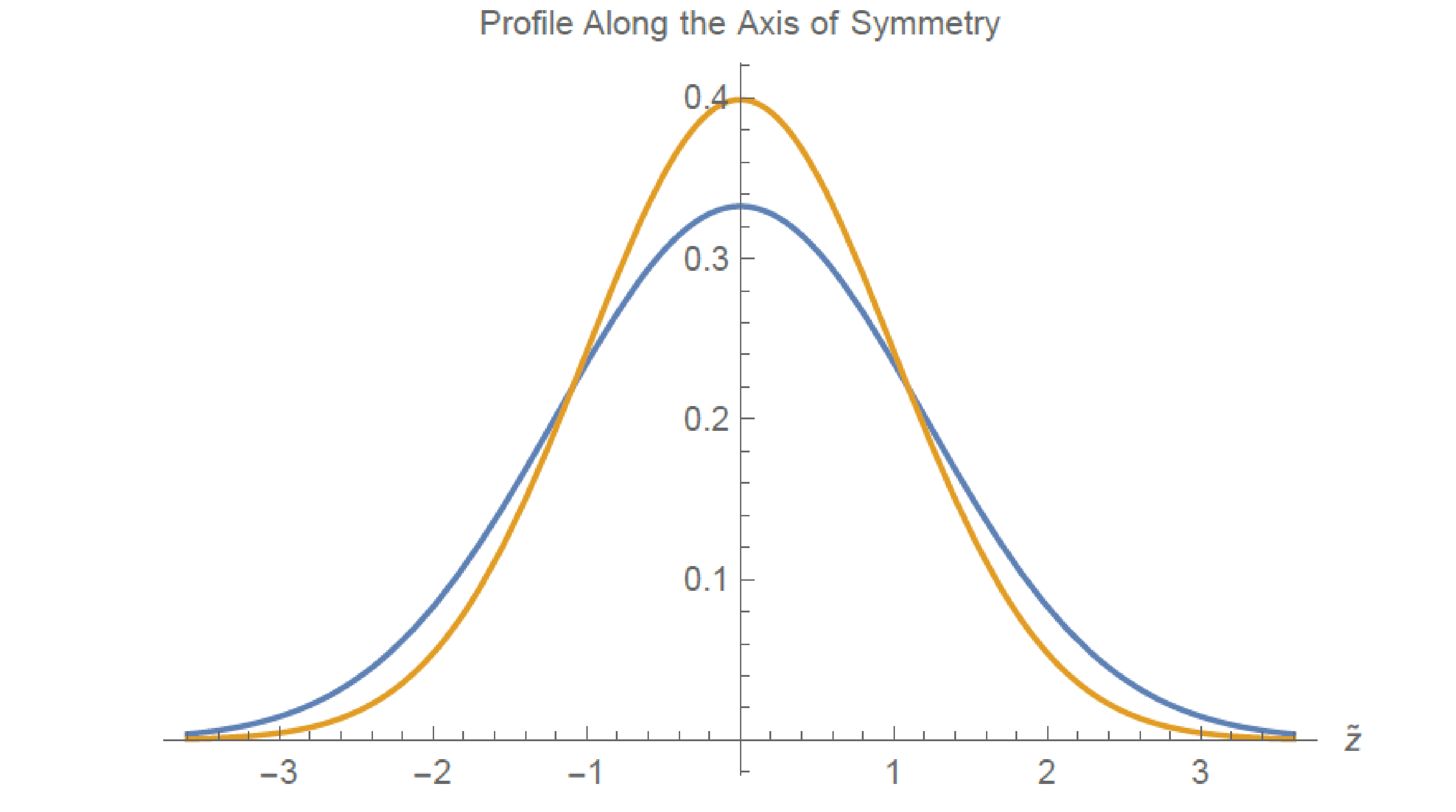
Using the non perturbative approach we can study the gravitational effect of the galactic matter and the IGM by considering both as one body with a time dependent density profile.

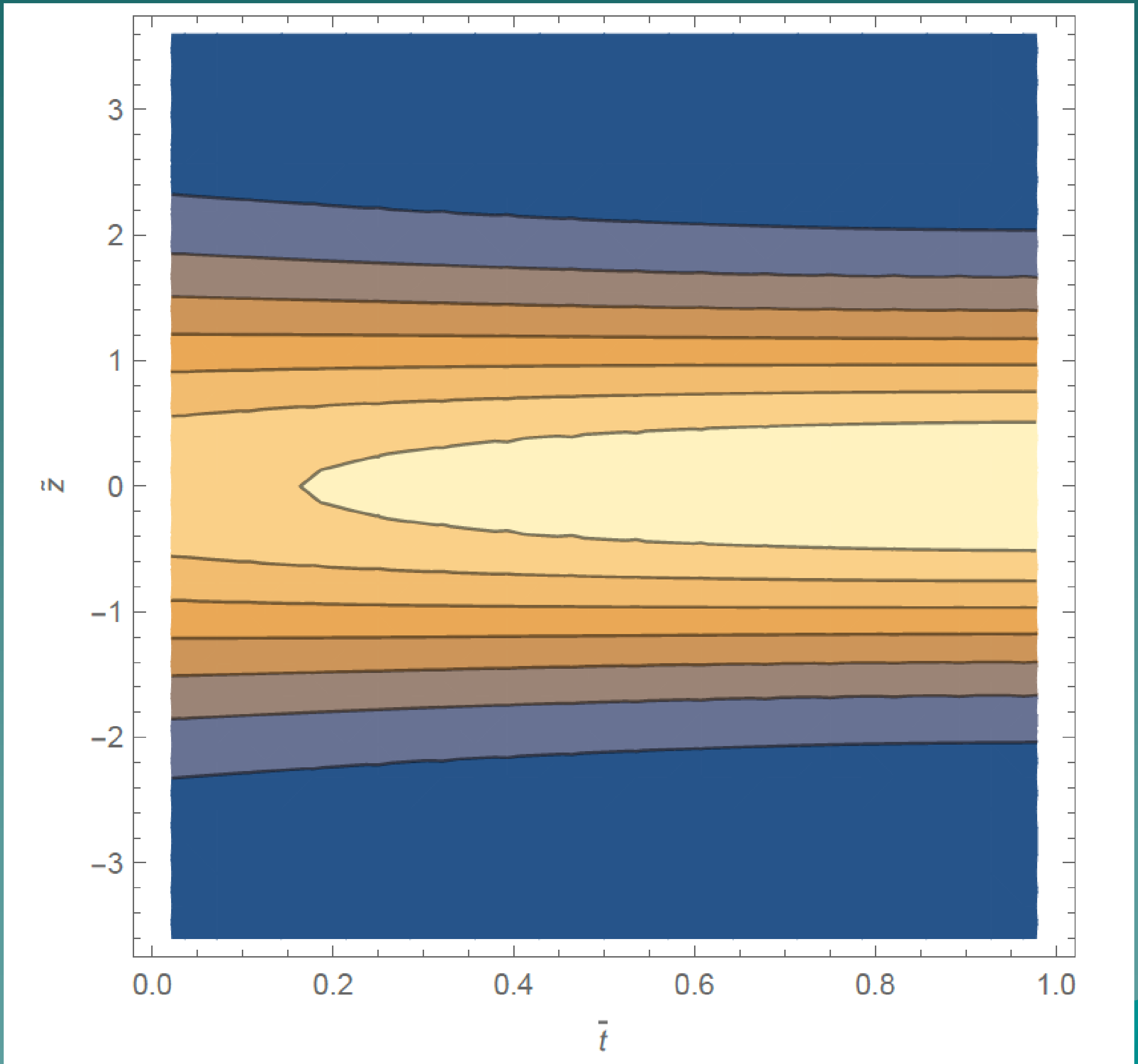
$$\rho = \rho_c \tilde{\rho}, \qquad \tilde{\rho} = \Sigma(\vec{x}_\perp) h(z,t)$$

$$h(z,t) = \frac{R_s}{\sqrt{2\pi}\sigma(t)} e^{-\frac{z^2}{2\sigma(t)^2}}.$$

$$\sigma(t) = \begin{cases} \sigma_i & \bar{t} \leq 0 \\ \sigma_i + (\sigma_f - \sigma_i)\bar{t}(2-\bar{t}) & 0 < \bar{t} < 1 \\ \sigma_f & \bar{t} \geq 1 \end{cases} \qquad \bar{t} \equiv \frac{t}{t_f}$$


$\tilde{\sigma}$
1.20
1.15
1.10
1.05
−1.0
−0.5
0.5
1.0
1.5
2.0
$\bar{t}$


Profile Along the Axis of Symmetry
0.4
0.3
0.2
0.1
−3
−2
−1
1
2
3
z̃


3
2
1
0
−1
−2
−3
z̃
0.0
0.2
0.4
0.6
0.8
1.0
t̄

$$\phi = -\frac{GM}{r}\psi_r$$

$$\phi_N = -\frac{GM}{r}\psi_N$$

$$\Delta\psi \equiv \psi_r - \psi_N \qquad \Rightarrow \qquad \lim_{r->\infty} \Delta\psi = 0$$

In other words, retardation is a near field effect.

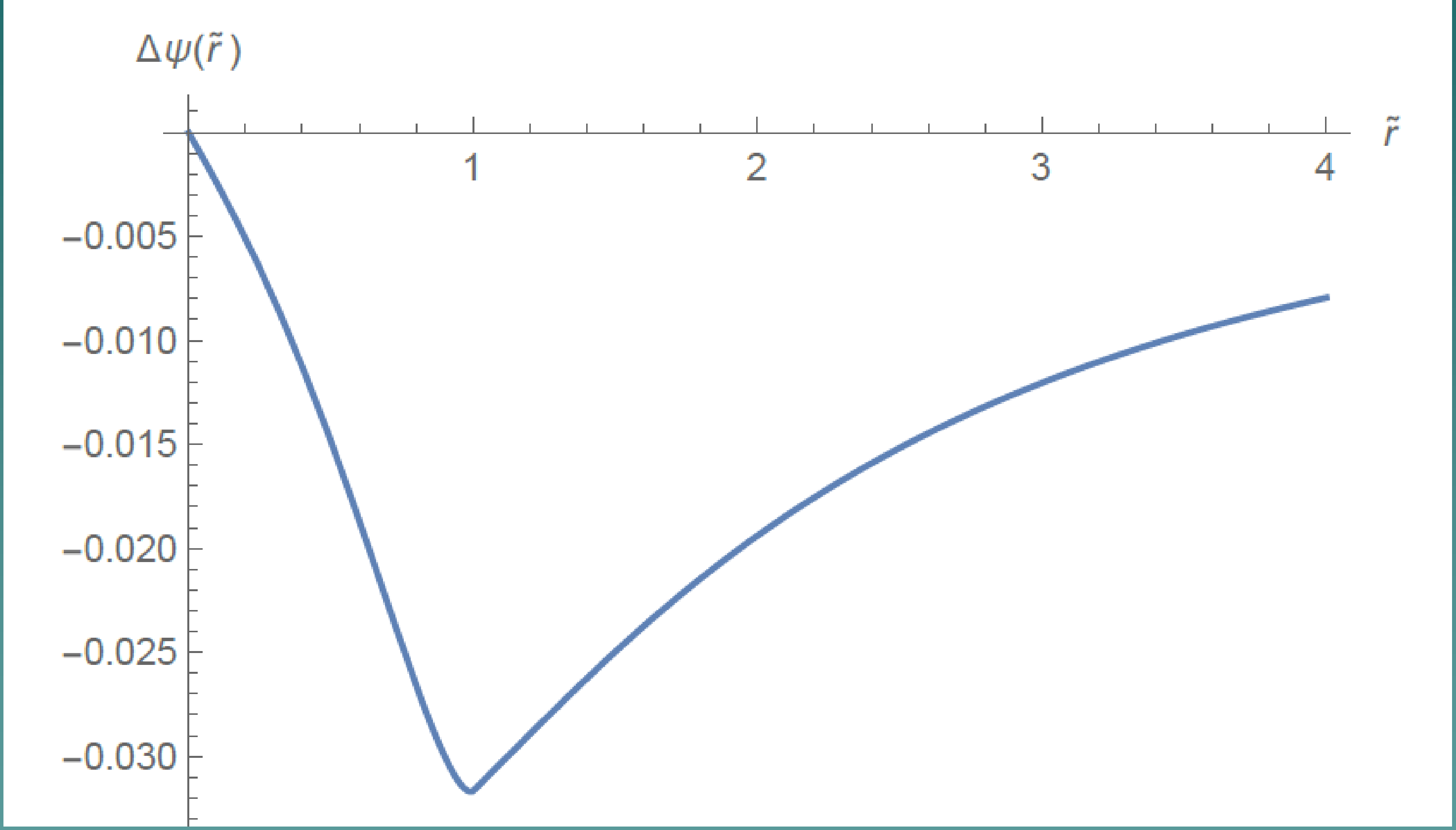
Why?

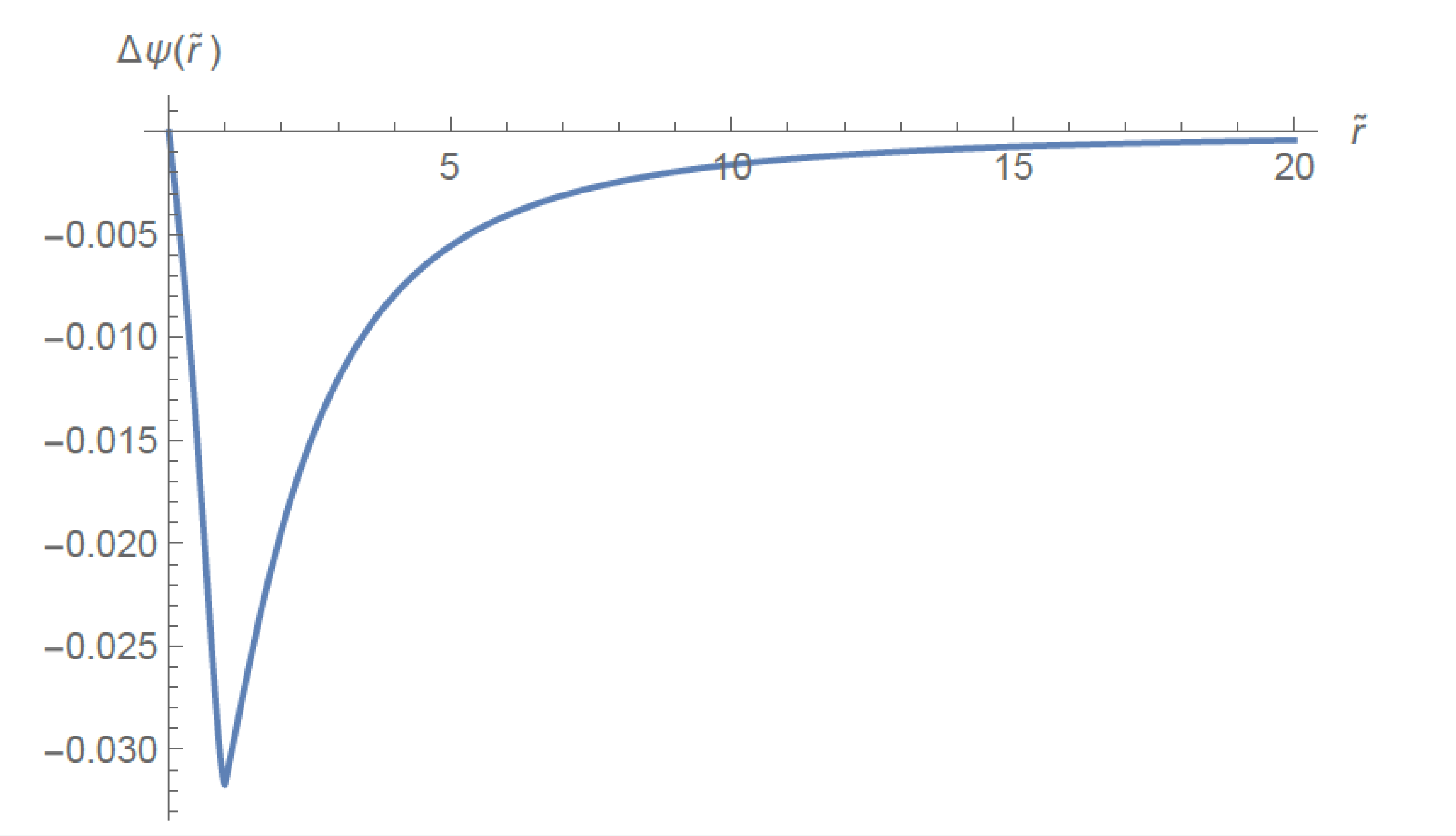
Let us look at the gravitational potential in the limit of large r, in which r is much bigger than a typical scale of the system: r>>Rs. In this limit, R is about the same as r.

$$\phi \simeq -G\int \frac{\rho(\vec{x}',t-\frac{r}{c})}{r}d^3x' = -\frac{G}{r}\int \rho(\vec{x}',t-\frac{r}{c})d^3x' = -\frac{GM}{r},$$

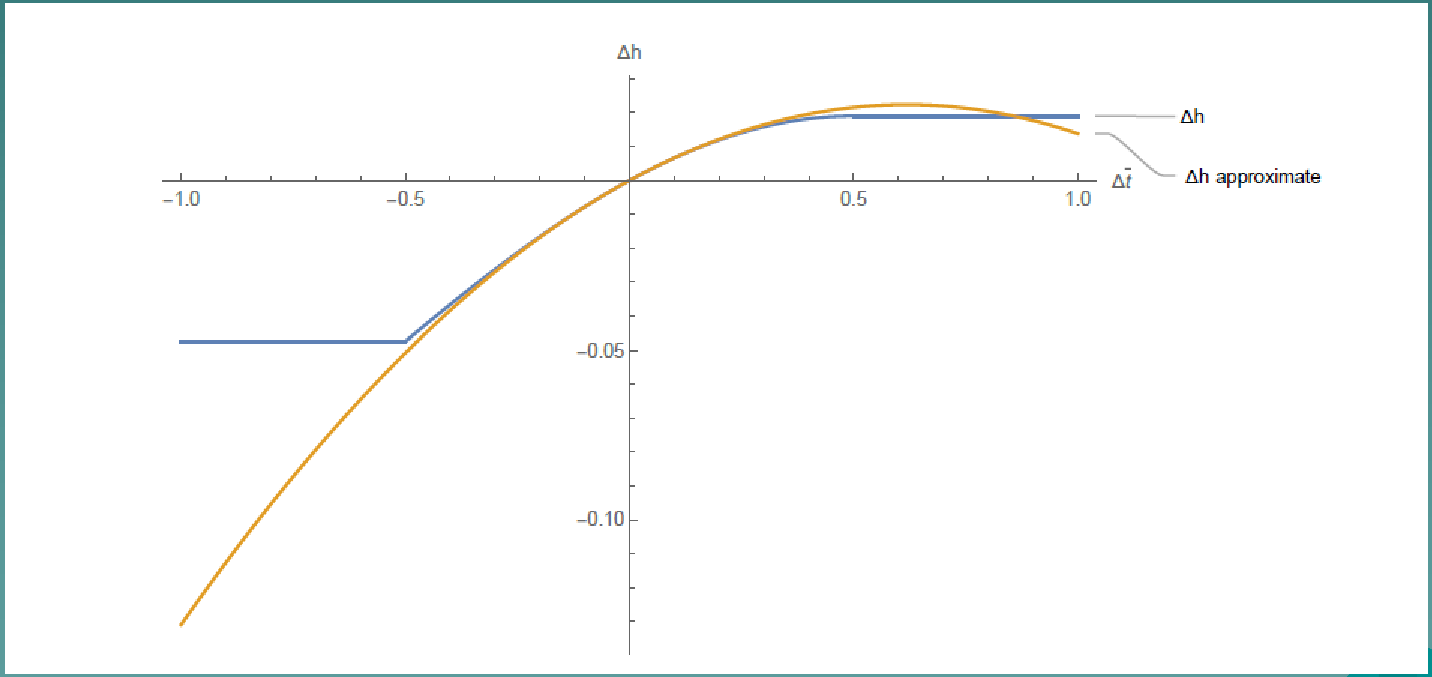
Which is the same as:

$$\phi_N \simeq -G\int \frac{\rho(\vec{x}',t)}{r}d^3x' = -\frac{G}{r}\int \rho(\vec{x}',t)d^3x' = -\frac{GM}{r}$$

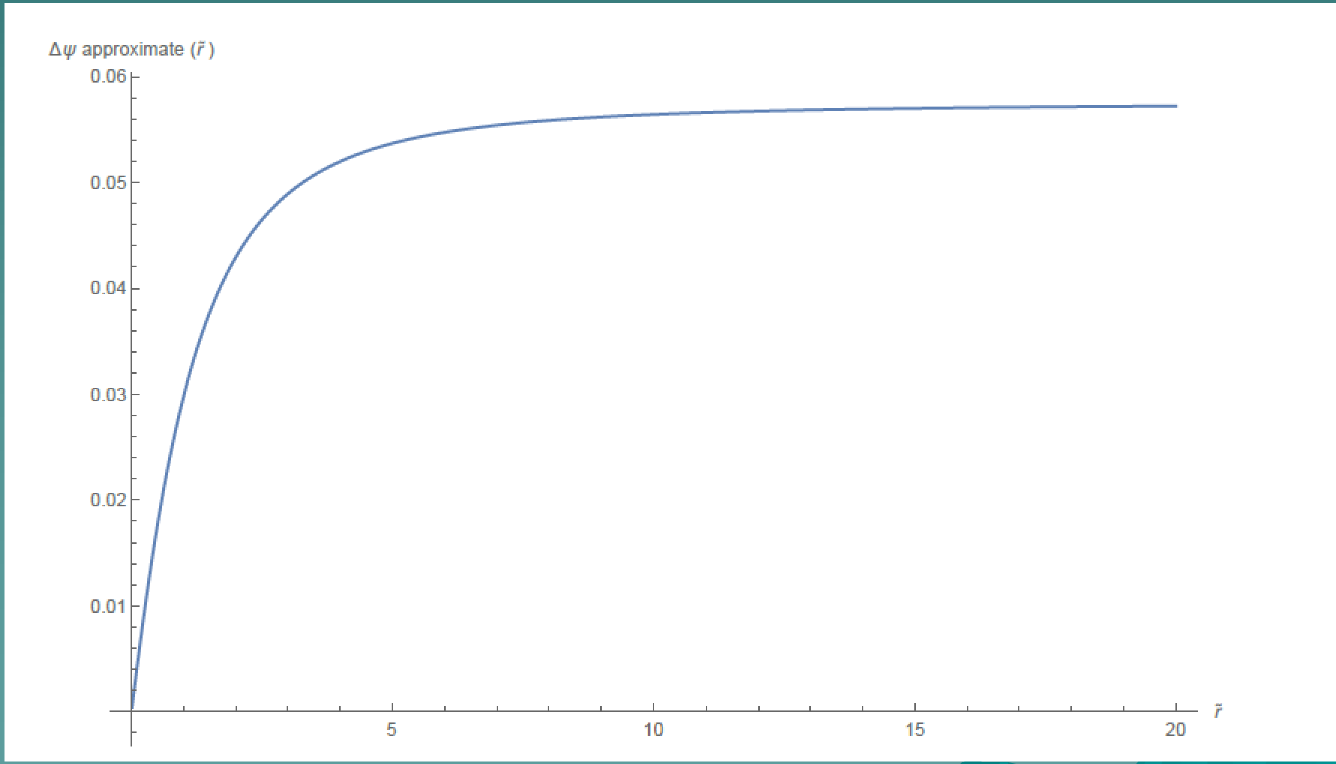

$\Delta\psi(\tilde{r})$
$\tilde{r}$
1
2
3
4
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030


$\Delta\psi(\tilde{r})$
$\tilde{r}$
5
10
15
20
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030

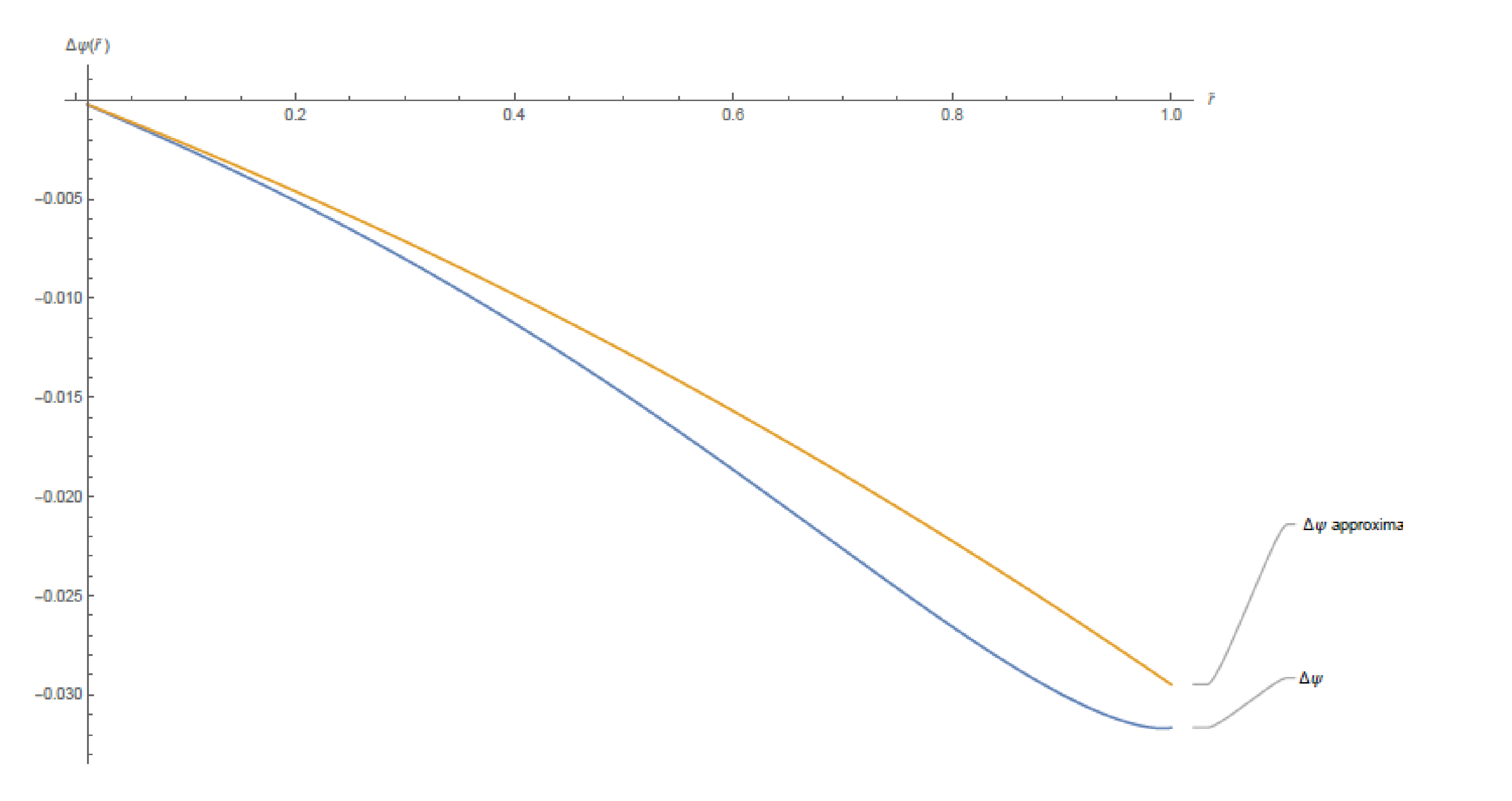
How good is the second order approximation?



Not very good, if one considers the galaxy + IGM



This is because the validity domain of the second order approximation has been violated. This can be amended by limiting the domain of integration to include only the galaxy.



MOND

Is an attempt (maybe the first attempt) to solve the standing problems of gravitation at the large scale not by postulating a new type of matter or particle but by looking more deeply at the phenomena of gravity (and gravity as Einstein taught is nothing but the structure of space-time).

MOND = MOdified Newtonian Dynamics

(or MOND = MOti's New Dynamics)

MOND

A theory of small acceleration defined by a typical acceleration a_0 in which:

$a \gg a_0$ is the Newtonian regime.

$a \ll a_0$ is the deep MOND regime.

In between there is an interpolation.

MOND & Retardation Theory

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

Standard Interpolation Function

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} \quad \Rightarrow \quad \mu(\frac{a}{a_0}) = \frac{1}{\sqrt{1+(\frac{a_0}{a})^2}}$$

Deep MOND Regime

$$a_0 \gg a, \ \mu \simeq \frac{a}{a_0}$$

$$a = \frac{v^2}{r}$$

$$\vec{F}_M = -\frac{G M a_0}{v^2 \, r} \hat{r}$$

Deep MOND Regime = Retardation Force becomes significant

$$|\ddot{M}| = \frac{2 M a_0 c^2}{v^2\ r}.$$

$$a_0 = 1.2 \times 10^{-10} \mathrm{ms}^{-2}$$

Problematic given a flat velocity as the left-hand side is spatial independent and the right-hand side depends on r, but can be used to calculate the mass secod derivative approximately.

The velocity at 15.33 kpc from the center of the M33 galaxy is 135,640 ms^{-1}.

We thus obtain $|\ddot{M}| \simeq 4.94 \times 10^{16}\ \mathrm{kgs}^{-2}$

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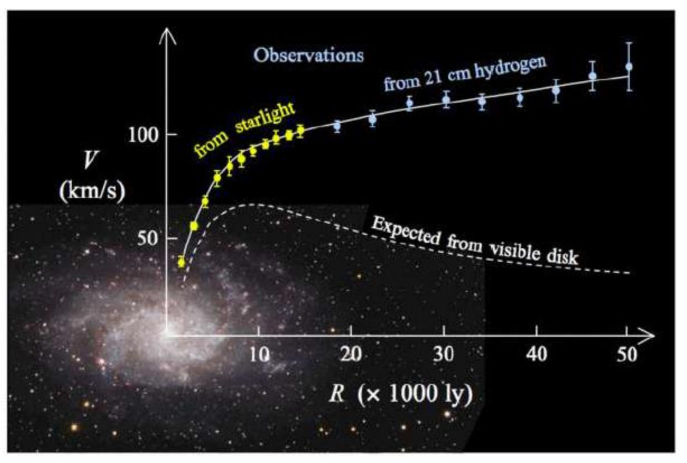
Parameter Estimation

$$t_r = \sqrt{\frac{M}{|\ddot{M}|}} \simeq 6.35\ 10^{11}\ \mathrm{s}$$



$$t_r \simeq 20{,}129 \text{ years}$$

$$R_r = c t_r \simeq 20,129 \text{ light years}$$

Retardation Theory as a MOND type of theory

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

Non-Standard Interpolation Function

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

$$\mu(x) = \frac{x^2}{\sqrt{1+x^4}} = \frac{1}{\sqrt{1+x^{-4}}} \quad \Rightarrow \quad \mu(\frac{a}{a_0}) = \frac{1}{\sqrt{1+(\frac{a_0}{a})^4}}$$

Deep MOND Regime

$$a_0 \gg a, \mu \simeq \left(\frac{a}{a_0}\right)^2$$

$$a = \frac{v^2}{r}$$

$$\vec{F}_M = -\frac{G M a_0^2}{v^4}\hat{r}$$

Deep MOND Regime

$$|\ddot{M}| = \frac{2 M a_0^2 c^2}{v^4}.$$

Tully-Fisher relation:

$$M = k v^4, \qquad k = \frac{|\ddot{M}|}{2 a_0^2 c^2}$$

a_0 calculated

$$|\ddot{M}| = \frac{2 M a_0^2 c^2}{v^4}.$$

$$a_0 = \frac{v^2}{c}\sqrt{\frac{|\ddot{M}|}{2M}}.$$

Retardation Theory is (approximately) a MOND type theory

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r} \simeq \vec{F}_N + \vec{F}_{ar}$$

But with a nonstandard interpolation function.

Conclusion

We show that coma cluster virial high velocities and galactic rotation curves, are explained in the framework of standard GR as effects due to retardation without assuming any exotic matter or modifications of the theory of gravity.

Conclusion

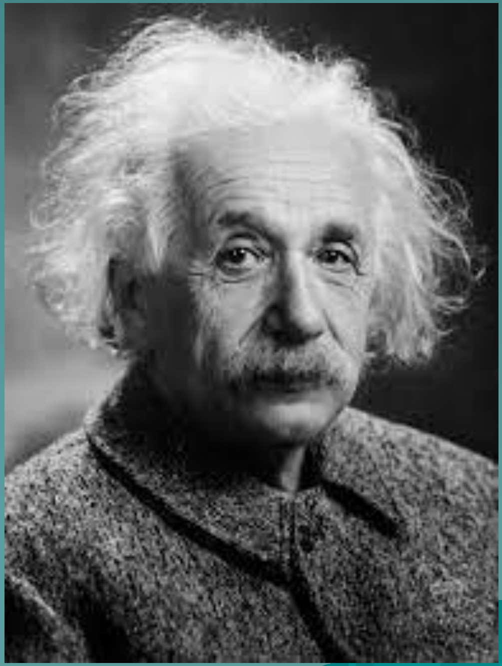
Retardation theory can be approximated as a MOND type theory with non-standard interpolation function.

What will happen if the mass outside the galaxy is totally depleted or not yet depleted? In this case retardation force should vanish. This was indeed reported for the galaxies NGC1052-DF2 and NGC 1052-DF4.

Pieter van Dokkum, Shany Danieli, Yotam Cohen, Allison Merritt, Aaron J. Romanowsky, Roberto Abraham, Jean Brodie, Charlie Conroy, Deborah Lokhorst, Lamiya Mowla, Ewan OSullivan & Jielai Zhang "A galaxy lacking dark matter" Nature volume 555, pages 629632 (29 March 2018) doi:10.1038/nature25767.

More Fundamental Conclusions

“**The Lord God is subtle**, **but malicious** he is **not**. ” — Albert Einstein. Remark during visit to Princeton University (1921) - “dark matter” effects are subtle indeed.




BEWARE THE DARK SIDE

Dark Energy - Cosmology

International Journal of Modern Physics D

Fitting of supernovae without dark energy

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arXiv:2207.14688v1 [astro-ph.CO] 28 Jul 2022

- ΛCDM, $\Omega_M = 0.287$ ($Q = 0.678$).
- FLRW with curvature, $\Omega_M = 0.355$, $\Omega_\Lambda = 0.835$ ($Q = 0.696$).
- Einstein–de Sitter with extinction ($Q = 0.453$).
- Linear Hubble–Lemaître law static Euclidean with extinction ($Q = 0.333$).
- Static Euclidean with tired light with extinction ($Q = 0.275$).
- FLRW with curvature and with evolution $\Omega_M = 0.957$, $\Omega_\Lambda \approx 0$ ($Q = 0.702$).
- Einstein–de Sitter with evolution $\Omega_M = 1$, $\Omega_\Lambda = 0$ ($Q = 0.709$).