Quantum field theory in cosmological spacetimes

Applications to early Universe physics

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Non-minimal scalar fields in cosmological spacetimes

1. Quantum field theory in cosmological spacetime
2. Gravitational Baryogenesis Model
3. Infrared distortion of quantum fluctuations from Inflation

Quantum field theory in cosmological spacetime

In the absence of a full quantum gravity theory: quantize matter field in a curved classical geometry

$$A_\mu, \phi, \psi \to \widehat{A}_\mu, \widehat{\phi}, \widehat{\psi}, \quad g_{ab} \to g_{ab}.$$

Example free scalar field

$$g^{ab}\nabla_a\nabla_b\phi + m^2\phi + \xi R\phi = 0, \quad [\widehat{\phi}, \widehat{\pi}] = i\hbar$$

In the context of cosmology it is useful to take advantage of the symmetries of FLRW spacetimes

$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2) \to \widehat{\phi}(\vec{x}, \tau) = a^{-1}\sum_{\vec{k}} h_k e^{i\vec{k}\vec{x}}\widehat{A}_{\vec{k}} + h_k^* e^{-i\vec{k}\vec{x}}\widehat{A}^\dagger_{\vec{k}}$$

where the modes h_k is harmonic oscillator

$$h_k'' + (k^2 + m^2 a^2 + (\xi - 1/6)a''/a)h_k = 0$$

Quantum field theory in cosmological spacetime

Quantization of scalar field in FLRW

$$\widehat{\phi}(\vec{x},\tau) = a^{-1} \sum h_k e^{i\vec{k}\vec{x}} \widehat{A}_{\vec{k}} + h_k^* e^{-i\vec{k}\vec{x}} \widehat{A}_{\vec{k}}^\dagger$$

Some features/obstacles of quantization in curved spacetimes:

1. There is no preferred positive/negative frequency splitting, no unique Poincare vacuum state, no ground state of Hamiltonian.
2. Vacuum expectation values (vev) of bilinear fields diverge. No normal ordering available.
3. Any quantum state yields non vanishing vev of the stress-energy tensor. semi-classical Einstein equation:

$$G_{ab} + \Lambda g_{ab} = \kappa \langle T_{ab} \rangle$$

QFTCS offer techniques to consistently address this issues.

Aim: incorporate these techniques for applications in cosmology.

CASE 1: GRAVITATIONAL BARYOGENESIS MODEL

Motivation: Baryogenesis

The manifest asymmetry of matter and antimatter is an open problem in current cosmology:

$$\eta \equiv \frac{n_B - n_{\overline{B}}}{s} = 2.74 \cdot 10^{-8} h^2 \Omega_B.$$

[Lambiase et al., 2013]

$$\eta^{\mathrm{CMB}} \sim (6.3 \pm 0.3) \cdot 10^{-10} \quad 0.0215 \leqslant \Omega_B h^2 \leqslant 0.0239$$

$$\eta^{\mathrm{BBN}} \sim (3.4 \pm 6.9) \cdot 10^{-10} \quad 0.017 \leqslant \Omega_B h^2 \leqslant 0.2024$$

Sakharov conditions

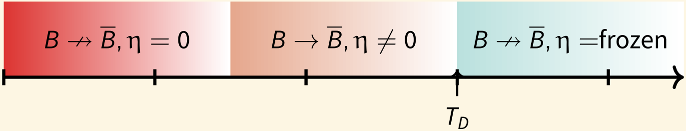
1. Process with baryon violation.
2. Charge *C* and Charge-Parity *CP* violation.
3. Departure from thermal equilibrium.

Gravitational Baryogenesis Model [Davoudiasl et al., 2004]

The GBM requires in thermal equilibrium

$$B \to \overline{B} \quad + \quad \frac{1}{M_*^2} J^a \nabla_a R \quad \Longrightarrow \quad \eta \approx \left. \frac{\partial_t R}{M_*^2 T} \right|_{T_D}$$

Observed value for $T_D \leqslant M_I$ and $M_* \leqslant M_P$ in radiation dominated era $T_D \leqslant R_D$. The new interaction acts as a "chemical potential".



Sakharov conditions

1. Process with baryon violation. ☺
2. Charge C and Charge-Parity CP violation. ☺
3. Departure from thermal equilibrium. 😅

Aim: non-minimally coupled (complex) scalar field with GBM

Complex Scalar field in cosmological spacetime

Assume a complex scalar filed and a flat FLRW spacetime

$$\left(g^{ab}D_aD_b + m^2 + \xi R\right)\widehat{\phi}(x) = 0 \quad D_a := \nabla_a + i\frac{1}{M_*^2}\nabla_a R$$

We can quantize the field in a FLRW $ds^2 = a^2(\tau)\left(d\tau^2 - dX^2\right)$

$$\widehat{\phi}(\vec{x},\tau) = \frac{1}{a}e^{-i\beta R}\sum_{\vec{k}}\left(B_{\vec{k}}h_k e^{ikx} + D^\dagger_{\vec{k}}h^*_k e^{-i\vec{k}\vec{x}}\right)$$

Observables of interest[a]

[a] ***Observation:*** j_a ***and*** T_{ab} ***do not depend on*** $\beta R'$

$$j_a = i(\phi^\dagger D_a\phi - (D_a\phi)^\dagger\phi) \quad T_{ab} = \frac{1}{2}D_a\phi D_b\phi^\dagger - \frac{1}{2}g_{ab}D^c\phi D_c\phi^\dagger + \ldots$$

$$\Phi^2(x,x') := \frac{1}{2}\left(\phi(x)\phi^\dagger(x') + \phi^\dagger(x')\phi(x)\right)$$

Quantum states: vacuum state

Given a base of modes $h_k e^{ikx}$ an the associated $B_{\vec{k}}, D^{\dagger}_{\vec{k}}$ The vacuum is defined as $B_{\vec{k}} |0\rangle = D_{\vec{k}} |0\rangle = 0$.

Problem: not a unique prescription. Possible candidate:

low energy states [Olbermann, 2007]

$$E_k[f] := \int d\tau \sqrt{-g} f^2 \rho_\omega[k] \quad \rho_\omega[k] = \left(\langle \omega | T_{ab} | \omega \rangle \right)_k u^a u^b$$

f compactly supported. Choose h_k such that it is minimizes.

Examples:

Minkowski: $h_k = \frac{1}{(2\sqrt{k^2+m^2})^{1/2}} e^{-ik\tau}$, Conformal: $h_k = \frac{1}{(2k)^{1/2}} e^{-ik\tau}$

However, for the GBM we need a thermal state and a notion of chemical potential!

Quantum states: KMS condition

In Minkowski (or static space time) we introduce a temperature and chemical potential as follows

> ***K(ubo)M(artin)S(chwinger) condition*** $\beta = T^{-1}$
>
> $$\left(\eta^{ab} D_a D_b + m^2\right) \phi(x) = 0 \quad D_a := \partial_a + \mu\delta_{a0}$$
>
> $$\langle KMS | \Phi^2(\vec{x}, \tau; \vec{x'}, \tau') | KMS \rangle = \langle KMS | \Phi^2(\vec{x}, \tau + i\beta; \vec{x'}, \tau') | KMS \rangle$$

$$\langle KMS | \Phi^2 | KMS \rangle = \langle vac | \Phi^2 | vac \rangle +$$

$$\sum_k \frac{e^{ik(x-x')}}{E_k} \frac{1}{e^{(E_k+\mu)\beta} - 1} + \frac{e^{-ik(x-x')}}{E_k} \frac{1}{e^{(E_k-\mu)\beta} - 1} \quad E_k = \sqrt{k^2 + m^2}$$

One can extract all possible information: relation with statistical mechanics [2].

Problem: Not suitable in time dependent backgrounds (FLRW).

[2] Equivalent to $Z = \mathrm{Tr}\, e^{\beta(H+\mu N)}$ in non relativistic case.

Quantum states: local thermal equilibrium (LTE) state

A local equilibrium thermal state proposal [Buchholz et al., 2002]

Local Thermal Equilibrium (LTE) states

For a given temperature distribution $\beta(x)$

1. Fix a set of thermal observables of order n: $\{\Phi^2(x), \partial_{\mu\nu}\Phi^2 \ldots\}$
2. Construct KMS states for a field ϕ_0 in Minkowski spacetime.
3. A LTE state is that for the set of 1. it equals the associated observables of state in 2. replacing $\beta \to \beta(x)$. Example:

$$\langle LTE| \, \Phi^2 \, |LTE\rangle = \langle KMS| \, \Phi_0^2 \, |KMS\rangle_{\beta(x)}$$

[Solveen, 2012] showed how to construct it for flat spacetime.

Not clear how to construct in FLRW spacetimes [Verch, 2012].

Not suitable for the GBM R' since it vanishes.

Quantum states: average (A)LTE state

Remember

$$E_k[f] := \int d\tau \sqrt{-g} f^2 \rho_\omega[k] \quad \rho_\omega[k] = (\langle \omega | T_{ab} | \omega \rangle)_k u^a u^b$$

It is time independent and minimum! Role of Hamiltonian in $Z = \mathrm{Tr}\, e^{\beta(H+\mu N)}$. Also can define

$$\mu Q_k[f] := \int d\tau \sqrt{-g} f^2 a^{-4} \frac{1}{M_*^2} R'$$

We can then define a field ϕ_A whose mode satisfy

$$\left((\partial_0 + \mu)^2 + E_k^2\right) g_k = 0$$

Average Local Thermal Equilibrium (LTE) states

For a given temperature distribution $\beta(x)$

1. Fix a set of thermal observables of order n: $\{\Phi^2(x), \partial_{\mu\nu}\Phi^2 \ldots\}$
2. Construct KMS states for a field ϕ_A in Minkowski spacetime.
3. A ALTE state is that for the set of 1. it equals the associated observables of state in 2. replacing $\beta \to \beta(x)$.

ALTE for the Gravitational Baryogenesis model

For general FLRW and scalar fields not trivial to find ALTE. But for GBM we can safely assume

1. $T > 100$GeV, ultra-relativistic particle $m = 0$ $\xi = 1/6$.
2. Focus on the decoupling time and assume R' almost constant.
3. Radiation era: $\beta(x) = T(\tau)^{-1} = a(\tau)T_0^{-1}$.

We find the ALTE for the GBM to be

$$\langle ALTE| : \Phi^2 : |ALTE\rangle = \frac{e^{-\frac{i}{M_*^2}(R(\tau)-R(\tau'))}}{a(\tau)a(\tau')} \sum_k \frac{e^{ik(x-x')}}{k} n_k + \frac{e^{-ik(x-x')}}{k} \overline{n}_k$$

with

$$n_k = \frac{1}{e^{(k+\mu)\beta} - 1} \quad \overline{n}_k = \frac{1}{e^{(k-\mu)\beta} - 1}$$

We can compute the baryon asymmetry number

$$n_B - n_{\overline{B}} \equiv u^a \langle ALTE| : j_a : |ALTE\rangle \approx \frac{R'(\tau_D)}{3a(\tau)M_*^2}$$

Bonus: Deviation from radiation dominated universe

Trace anomaly prevents a pure radiation dominated universe with $R' \neq 0$. The semiclassical Einstein equation

$$R = 8\pi G_N : \langle T_a^a \rangle :$$

Using our ALTE state we obtain

$$\langle : T_0^0 : \rangle = \frac{\pi^2}{30} T^4 + \frac{R'(\tau_D)}{4a^2 M_*} T^2 + \frac{1}{2880\pi^2} {}^{(3)}H_0^0$$

$$\langle : T_i^i : \rangle = -\frac{\pi^2}{90} T^4 - \frac{R'(\tau_D)}{12a^2 M_*} T^2 + \frac{1}{2880\pi^2} {}^{(3)}H_i^i$$

which yields (including all type of matter fields)

$$R = -8\pi G_N \frac{N_s + 11N_f + 6N_v}{2880\pi^2} \left(R_{ab} R^{ab}\right)$$

and the baryon asymmetry is computed in (almost) radiation

$$\eta \approx \frac{720\pi^2}{g_*} k_3 \sqrt{\frac{g_*^5}{90^5}} \frac{T^9}{M_p^7 M_*}$$

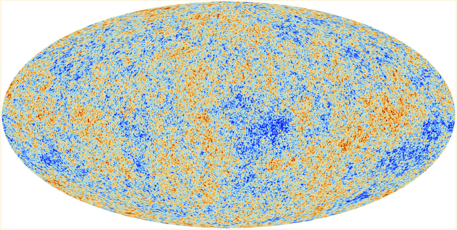
recovers the observed result for values $T_D \leqslant M_I$ and $M_* \leqslant M_P$! ☺

CASE 2: Generation of Quantum Fluctuations in de-Sitter

Quantum fluctuations from de-Sitter

Quantum fluctuations are created in expanding universe (Parker, 1968).

An inflationary de-Sitter universe strong enough to generated classical inhomogeneities for structure formation.



Typical observables we need to compute:
$\langle \phi^2 \rangle$, $\langle T_{ab} \rangle$, etc.

These are ultraviolet divergent and need a proper regularisation mechanism

Expectation values of observables

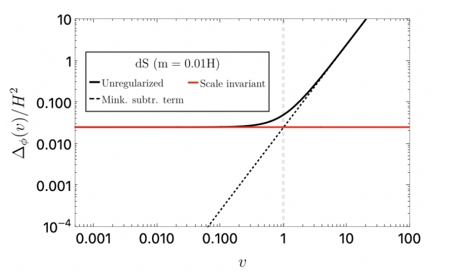
Assume a (real) scalar field ($\xi = 0$) in de Sitter spacetime $a \sim e^{Ht}$.

Quantization of scalar field in FLRW

$$\widehat{\phi}(\vec{x}, \tau) = \sum h_k e^{i\vec{k}\vec{x}} \widehat{A}_{\vec{k}} + h_k^* e^{-i\vec{k}\vec{x}} \widehat{A}_{\vec{k}}^\dagger$$

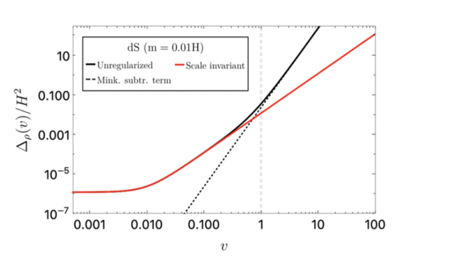
The vacuum state is the Bunch-Davies state. The v.e.v of two point function and the energy density in that state are

$$\langle \phi^2 \rangle = \int d^3k \Delta_\phi, \quad \Delta_\phi = |h_k|^2; \quad h_k = \frac{-i\sqrt{\tau\pi}}{2a} H^{(1)}_{\sqrt{9/4 - m^2}}(-k\tau)$$



Expectation values of observables

$$\rho \equiv \langle T_{ab} u^a u^b \rangle = \int d^3k \left[|h_k'|^2 + (k^2 + a^2 m^2) |h_k|^2 \right]$$



It is easy to check that these integral diverge in the limit $k \to \infty$!

Use adiabatic Regularization!

Introduction to Adiabatic Regularisation

- Regularise: detect the divergent part and eliminate it in a meaningful way. E.g. scalar auto-energy QED

$$\Gamma_2^{\mu\nu} = ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{-4k^\mu k^\nu + 2g^{\mu\nu}((p-k)^2 - m^2)}{((p-k)^2 - m^2 + i\epsilon)(k^2 - m^2 + i\epsilon)}$$

- Dimensional regularisation in perturbative scalar QED in Minkowski spacetime

$$\Gamma_2^{\mu\nu} = \frac{-e^2}{8\pi^2}(p^2 g^{\mu\nu} - p^\mu p^\nu)\int_0^1 dx(2x-1)\left(\frac{2}{\epsilon} + \text{finite}\right)$$

- Divergences can always be reabsorbed in finite counter-terms.
- Infinite ways of subtracting the finite part: once fixed the experiment, subtraction scheme is fixed. E.g. electric charge

Introduction to Regularisation

- Regularisation of observables, e.g. the two point function (covariantly, locally):
- Different methods proposed. They can differ by:

$$\langle : \phi^2 : \rangle - \langle : \overline{\phi}^2 : \rangle = \alpha m^2 + \beta R$$

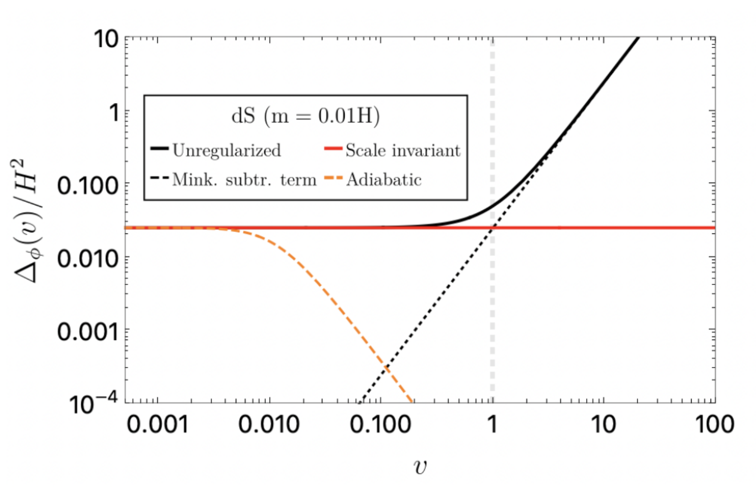
- Adiabatic regularization (Parker-Fulling, 1974) subtracts directly the power spectrum

$$\langle : \phi^2 : \rangle = \int_0^\infty dlnk(\Delta_\phi - \mathcal{Q}_k))$$

- It is constructed by a WKB type expansion of the mode solutions h_k in of time derivatives

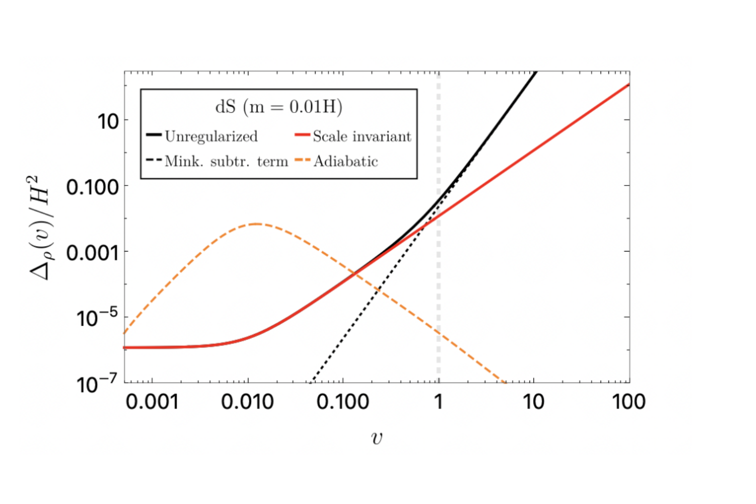
$$\mathcal{Q}_k = \frac{1}{2a^2\omega_k} - \frac{(\xi - 1/6)R}{4\omega_k^3}, \quad \omega_k = \sqrt{k^2 + m^2a^2}$$

Adiabatic regularisation for Quantum fluctuation



Potential change of standard observable predictions for slow-roll inflation: [Agullo, Navarro-Salas, Olmo, Parker- PRL 2008, 2009]

Adiabatic regularisation for Quantum fluctuations



Physical Scale Adiabatic Regularization (PSAR)

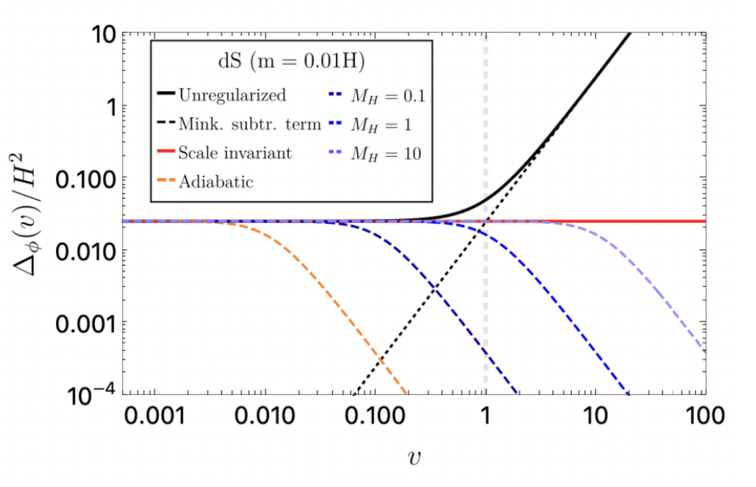
- Different methods can differ by:

$$\langle : \phi^2 : \rangle - \langle : \overline{\phi}^2 : \rangle = \alpha m^2 + \beta R$$

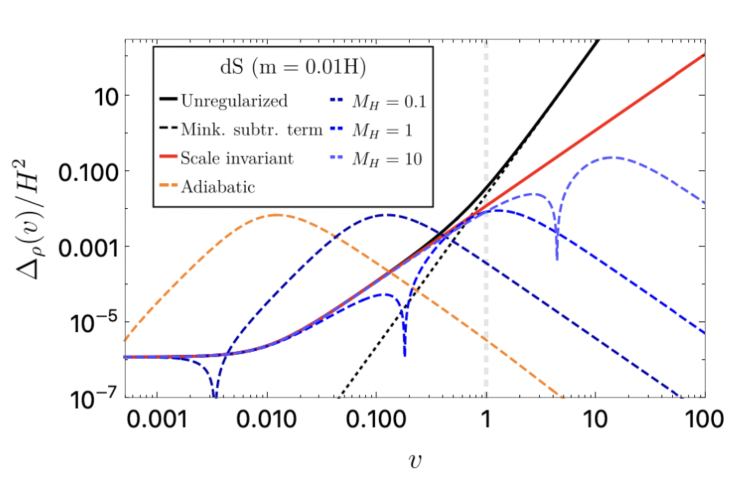
- For $\langle : T_{ab} : \rangle$ three arbitrary parameters.
- Adiabatic regularisation can be extended to include these arbitrary parameters. (AF & J Navarro-Salas, 2019: A F & Torrenti, 2023): **Physical Scale Adiabatic Regularization**
- It is based on a modification of the WKB ansatz
- For the two point function

$$\mathcal{Q}_k = \frac{1}{2a^2\omega_k(m)} - \frac{(\xi - 1/6)R}{4\omega_k^3(M)}, \quad \omega(M)_k = \sqrt{k^2 + M^2 a^2}$$

PSAR for Quantum fluctuations



PSAR for Quantum fluctuations



Renormalization conditions

To connect with observations we need to fix the couplings at a given renormalization point. The semiclassical Einstein equations

$$\frac{1}{8\pi G}R - 4\Lambda - \alpha\Box R = \langle :T^a_a:\rangle_M; \quad \nabla_a\langle :T^{ab}:\rangle = 0$$

Idea: fix scale $M = m$, for $R \ll m^2$, can construct $\langle :T^a_a:\rangle_m$

$$\frac{1}{8\pi G_0}R - 4\Lambda_0 - \alpha_0\Box R = \mathcal{O}(m^{-2})$$

Back to the scale of inflation

$$\frac{1}{8\pi G_0}R - 4\Lambda_0 - \alpha_0\Box R = \langle :T^a_a:\rangle_m;$$

Problem: We still want to use $\langle :T^a_a:\rangle_M$ and not $\langle :T^a_a:\rangle_m$.

Running couplings

The difference between $\langle : T^a_a : \rangle_M$ and $\langle : T^a_a : \rangle_m$ can be reabsorbed in Λ, G, and α s.t. the semiclassical EE are independent of M We want to go from

$$\frac{1}{8\pi G_0} R - 4\Lambda_0 - \alpha_0 \Box R = \langle : T^a_a : \rangle_m;$$

to

$$\frac{1}{8\pi G_M} R - 4\Lambda_M - \alpha_M \Box R = \langle : T^a_a : \rangle_M;$$

the difference between couplings is

$$\Lambda_M = \Lambda_0, \quad \alpha_M = \alpha_0 - \frac{1}{32\pi^2} \log\left(\frac{m^2}{M^2}\right)$$

$$G_M = \frac{G_0}{1 + \frac{G_0}{12\pi}\left(m^2 - \mu^2 + m^2 \log\left(\frac{m^2}{M^2}\right)\right)}$$

Running couplings

$$G_M = \frac{G_0}{1 + \frac{G_0}{12\pi}\left(m^2 - \mu^2 + m^2 \log\left(\frac{m^2}{M^2}\right)\right)}$$

Quadratic corrections are present in the Asymptotic Safe approach to quantum gravity (Reuter, 2008; Sauerressig 2023)

$$G(k) = \frac{G(k_0)}{1 + \omega G(k_0)\left(k^2 - k_0^2\right)}$$

Possible universality of running of the Newton constant

Conclusions

1. Quantum field theory in curved spacetime has accumulated many techniques to address the issues not present in flat QFT.
2. To gain a good understanding of our early universe we require these techniques.
3. It can lead to new insights and physics that could potentially impact current and future measurements.

References

Buchholz, D., Ojima, I., and Roos, H. (2002).
Thermodynamic properties of nonequilibrium states in quantum field theory.
Annals Phys., 297:219–242.

Davoudiasl, H., Kitano, R., Kribs, G. D., Murayama, H., and Steinhardt, P. J. (2004).
Gravitational baryogenesis.
Phys. Rev. Lett., 93:201301.

Lambiase, G., Mohanty, S., and Prasanna, A. R. (2013).
Neutrino coupling to cosmological background: A review on gravitational Baryo/Leptogenesis.
Int. J. Mod. Phys. D, 22:1330030.

Olbermann, H. (2007).
States of low energy on Robertson-Walker spacetimes.
Class. Quant. Grav., 24:5011–5030.

Solveen, C. (2012).