IARD 2024 Meeting

The 14th Biennial Conference on Classical and Quantum Relativistic Dynamics of Particles and Fields

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Time dispersion in Bound States

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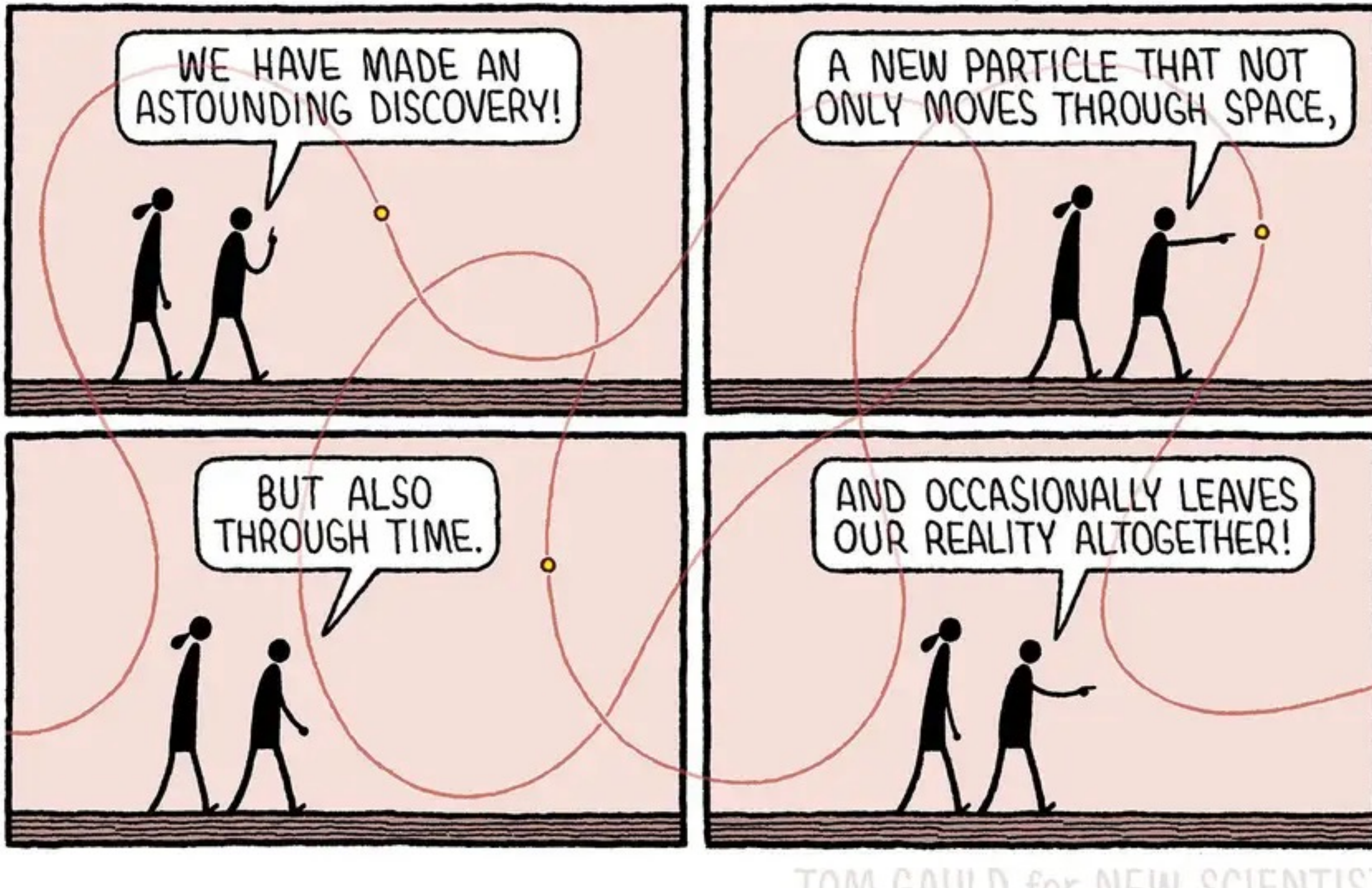
"We are trying to prove ourselves wrong as quickly as possible, because only in that way can we find progress."
— R P Feynman

Should quantum mechanics be applied along the time dimension, Exactly as it is in space?

- Not well-defined
- Should have been seen, if only by chance
- Current theory fully covariant
- Current theory fully confirmed

Paths in time



https://www.newscientist.com/article/2424379-tom-gauld-on-an-astounding-discovery/

Paths and path integrals

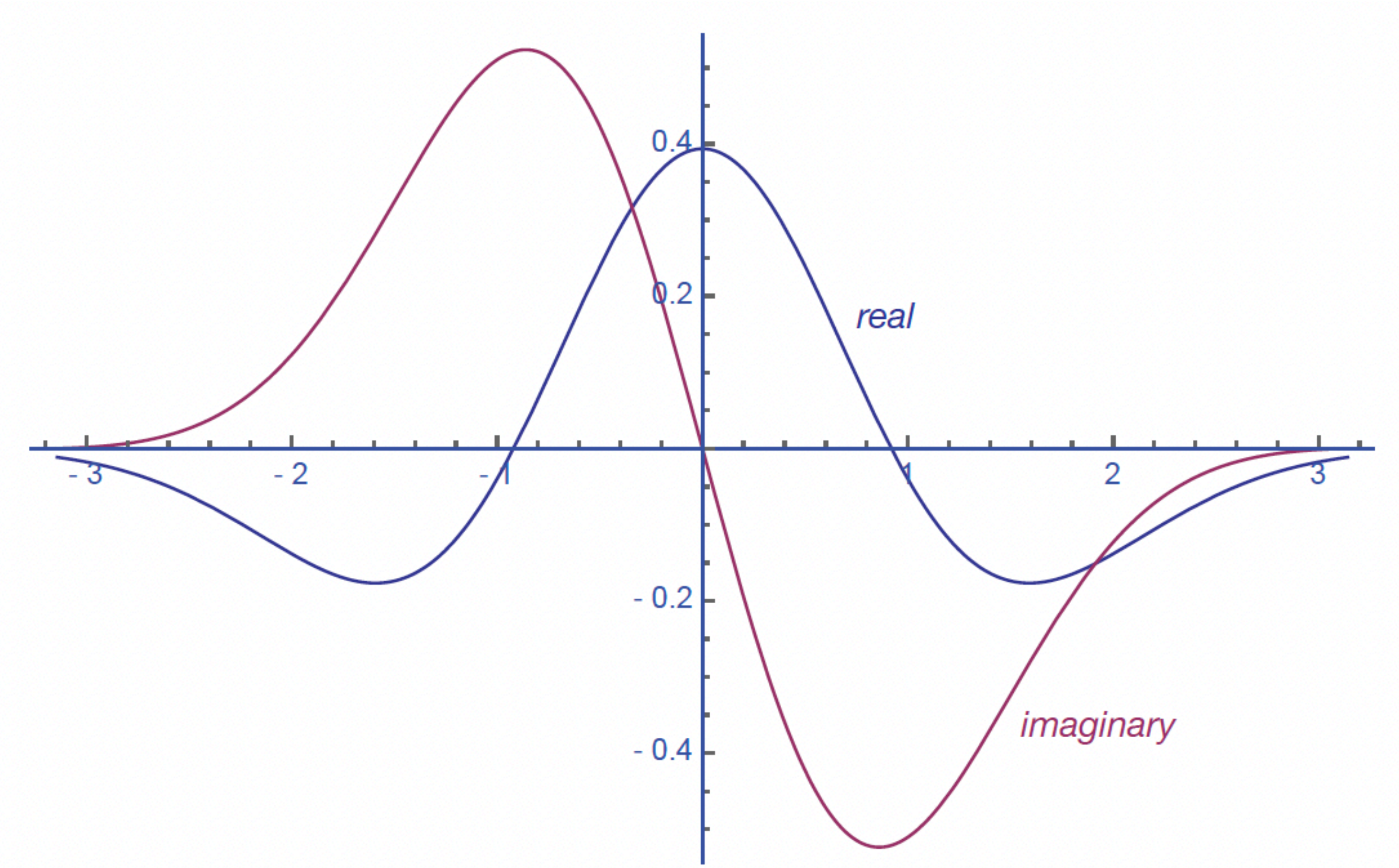
$$\psi_T\left(x_T\right) = \int \mathcal{D}x_\tau \exp\left(\imath \int_0^T d\tau L\left[x, \dot{x}\right]\right)\psi_0\left(x_0\right)$$

$$\mathcal{D}\vec{x} \equiv \prod_{n=0}^{N} d\vec{x}_n \quad \Rightarrow \quad \mathcal{D}x \equiv \prod_{n=0}^{N} dt_n d\vec{x}_n$$

$$\mathcal{L}\left[x_\tau, \dot{x}_\tau\right] = -\frac{1}{2}m\dot{x}^\mu \dot{x}_\mu - q\dot{x}^\mu A_\mu\left(x\right) - \frac{m}{2}$$

- Extend paths in space to include motion in time
- Keep everything else the same
- See what breaks

Convergence



- Usual tricks violate covariance
- Use Morlet wavelet analysis instead
- No loss of generality

Feynman/ Stueckelberg equation

$$-2m\imath\frac{\partial\psi_\tau}{\partial\tau}=\left(\left(p_\mu-qA_\mu\right)\left(p^\mu-qA^\mu\right)-m^2\right)\psi_\tau$$

Clock time

- Short time limit of path integrals
- Specialization of the time parameter (Fanchi's historical time)

Short and long time limits

Klein-Gordon:

$$0 = \left(\left(E - A_0 \right)^2 - \left(\vec{p} - \vec{A} \right)^2 - m^2 \right) \psi_\tau, E \Leftrightarrow \imath \frac{\partial}{\partial t}$$

Schrödinger:

$$\imath \frac{\partial \psi_\tau}{\partial \tau} = \frac{\left(\vec{p} - \vec{A}^2 \right)}{2m} \psi_\tau + q\Phi\psi_\tau, \vec{p} \equiv -\nabla, \Phi \equiv A_0, E \approx m$$

Energy scales:

$$\frac{\vec{p}^2}{2m} \sim eV \qquad \varpi_p \sim \frac{eV^2}{MeV} \sim 10^{-6} eV$$

Limits of SQM

$$\phi_\tau^S(\vec{x}) = \sum_{\vec{k}} \frac{1}{\sqrt{2V\omega_{\vec{k}}}} a_{\vec{k}} \mathrm{e}^{-\imath\omega_{\vec{k}}\tau + \imath\vec{k}\cdot\vec{x}} + \sum_{\vec{k}} \frac{1}{\sqrt{2V\omega_{\vec{k}}}} a_{\vec{k}}^\dagger \mathrm{e}^{\imath\omega_{\vec{k}}\tau - \imath\vec{k}\cdot\vec{x}}$$

Normalization

$$\imath\Delta_{xy}^S(\vec{x}-\vec{y}) = \langle 0|T\left\{\phi_x^S(\vec{x}), \phi_y^S(\vec{y})\right\}|0\rangle$$

$$\frac{e^{-\imath\omega_{\vec{k}}\tau}}{2\omega_{\vec{k}}}, \omega_{\vec{k}} \equiv \sqrt{\vec{k}^2 + \mu^2} \Rightarrow \int d\omega \frac{e^{-\imath\omega\tau}}{\omega^2 - \vec{k}^2 - \mu^2 + \imath\epsilon}$$

$$\lim_{\tau \to \pm\infty} \Rightarrow \delta\left(\sum_i \omega_i\right)$$

Extending QED in time

$$\imath m \frac{\partial}{\partial \tau} \to \imath E \frac{\partial}{\partial \tau}$$

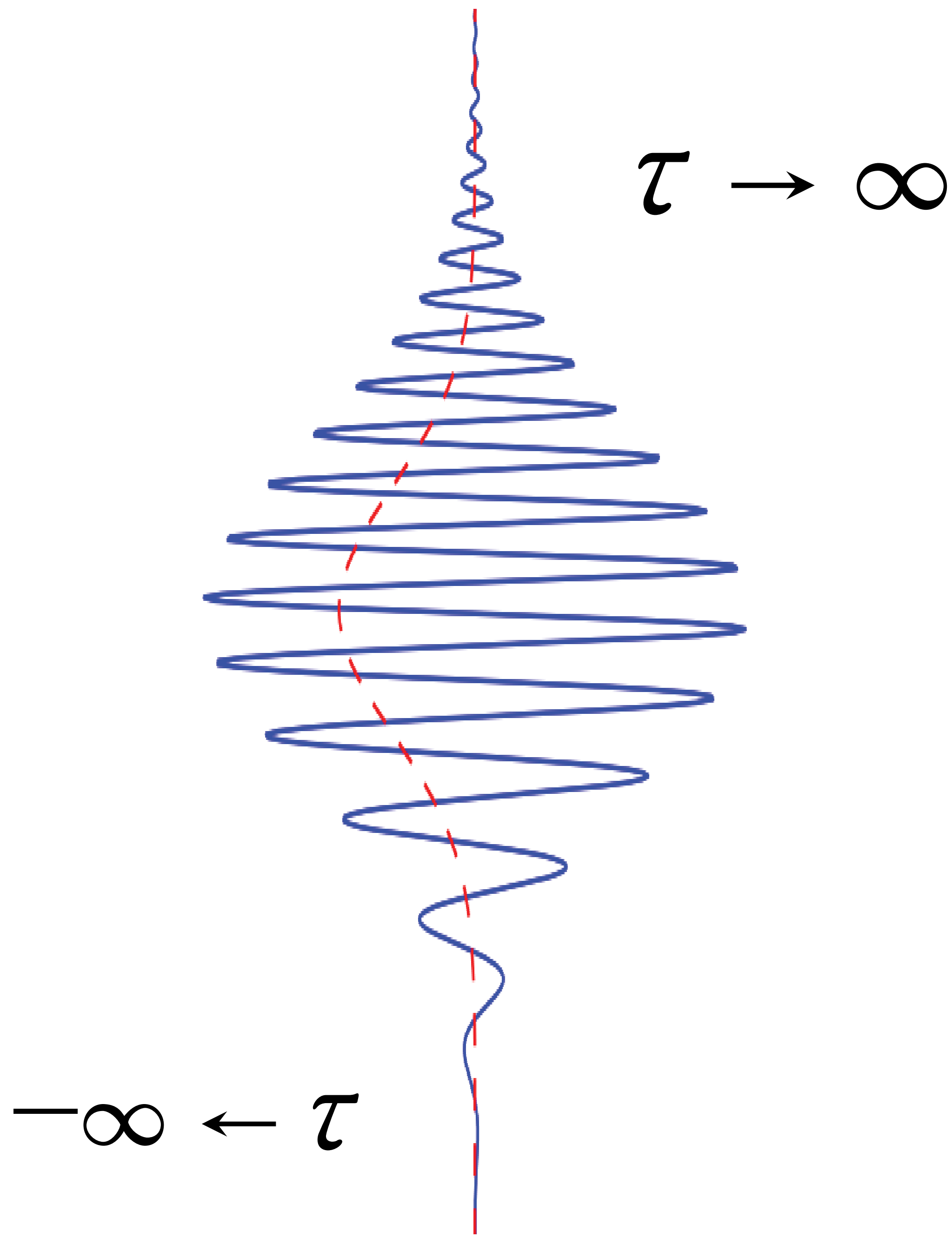
$$\imath \frac{\partial}{\partial \tau} \psi_\tau = -\frac{p^2 - m^2}{2E} \psi_\tau \quad \longrightarrow \quad \psi_\tau = \exp(-\imath \varpi_p \tau) \psi_0$$

$$\varpi_p \equiv -\frac{E^2 - \vec{p}^2 - m^2}{2E}$$

$$u_1^{(SQM)} = \sqrt{\frac{E_{\vec{p}} + m}{2m}} \begin{pmatrix} 1 \\ 0 \\ \frac{p^3}{E_{\vec{p}} + m} \\ \frac{p^1 + \imath p^2}{E_{\vec{p}} + m} \end{pmatrix} \quad \longrightarrow \quad u_1^{(TQM)} = \sqrt{\frac{E + m}{2m}} \begin{pmatrix} 1 \\ 0 \\ \frac{p^3}{E + m} \\ \frac{p^1 + \imath p^2}{E + m} \end{pmatrix}$$

$$-\imath \gamma_0 \frac{\partial}{\partial \tau} \psi_\tau = (\not{p} - m) \psi_\tau$$

Differences



- Incoming particles have to be normalizable
- Can handle short times: get conservation of 4 momentum even at short times
- Ultraviolet (UV) divergences contained by combination of entanglement in time and dispersion in time

Clock time coordinate time

$$\tau \equiv \left\langle \psi^{(Lab)} \middle| t^{(operator)} \middle| \psi^{(Lab)} \right\rangle$$

$$E\left(\imath \frac{\partial}{\partial \tau}\right) \rightarrow p_0 P_0^{(Lab)} \rightarrow p_\mu P^{(Lab)\mu}$$

- Clock time as expectation of coordinate time
- Can write covariantly
- And invariantly
- Minor effect in practice

Entropic Estimate

$$\Delta E \equiv \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$$

$$\varphi_0(E) = \sqrt[4]{\frac{1}{\pi \sigma_E^2}} e^{\imath (E - E_{nlm})\tau - \frac{(E-E_{nlm})^2}{2\sigma_E^2}} \qquad \sigma_E^2 \equiv \frac{(\Delta E)^2}{2}$$

$$\Delta E \equiv \sqrt{\langle m^2 + |\vec{p}|^2 \rangle - \left\langle \sqrt{m^2 + |\vec{p}|^2} \right\rangle^2} \approx \frac{1}{2m} \sqrt{\langle |\vec{p}|^4 \rangle - \langle |\vec{p}|^2 \rangle^2}$$

$$\Delta E_{100} \approx \frac{1}{2m} \sqrt{\frac{5}{a_0^2} - \frac{1}{a_0^2}} = \frac{1}{2m} \frac{2}{a_0^2} = \frac{1}{2} \frac{|\vec{p}|^2}{2m} = 3.3 eV$$

$\hbar = 6.582\, 10^{-16} eVs \rightarrow \Delta t_{100} \approx$ $200as$ → $43as$ shortest pulse

Massive particle Propagator

$$K_\tau(p;p') = \exp(-\imath\varpi_p\tau)\delta^4(p-p')\theta(\tau)$$

$$\varpi_p \equiv -\frac{E^2-\vec{p}^2-m^2}{2E}$$

$$E \approx m$$

$$K_\tau(x;x') = -\imath\frac{m^2}{4\pi^2\tau^2}e^{-\imath m\frac{(t-t')^2}{2\tau}+\imath m\frac{(\vec{x}-\vec{x}')^2}{2\tau}-\imath\frac{m}{2}\tau}\theta(\tau)$$

$$E > m, \frac{\delta E}{\bar{E}} \ll 1$$

$$K_\tau(x;x') = -\imath\frac{\bar{E}^2}{4\pi^2\tau^2}e^{-\imath\bar{E}\frac{(t-t')^2}{2\tau}+\imath\bar{E}\frac{(\vec{x}-\vec{x}')^2}{2\tau}-\imath\frac{m}{2}\tau}\theta(\tau)$$

Photon propagator

$$K_\tau(k) = e^{-i\tau\varpi_k}\Theta(\tau), \varpi_k \equiv -\frac{w^2-\varkappa^2}{2w}, \varkappa \equiv \vec{k}^2$$

$$K_\tau(w,\vec{k}) = \exp\left(\imath\frac{w}{2}\tau - \imath\frac{\varkappa^2}{2w}\tau\right)\Theta(\tau)$$

$$\frac{1}{2}\sqrt{\pi}\,\varkappa\sqrt{\tau}\left|\frac{1}{\left(t-\frac{\tau}{2}\right)^{3/2}(2t+\tau)}\right|\left((\tau-2t)\left|t+\frac{\tau}{2}\right|-2\left|t^2-\frac{\tau^2}{4}\right|\right)J_1\left(\sqrt{2}\,\varkappa\sqrt{\tau}\sqrt{\left|t-\frac{\tau}{2}\right|}\right)$$

Photon propagator by $\varkappa \equiv |\vec{k}|$

$$w = \kappa + \delta w$$

$$\delta w = \alpha \kappa$$

$$\alpha \equiv \frac{w - \kappa}{\kappa}$$

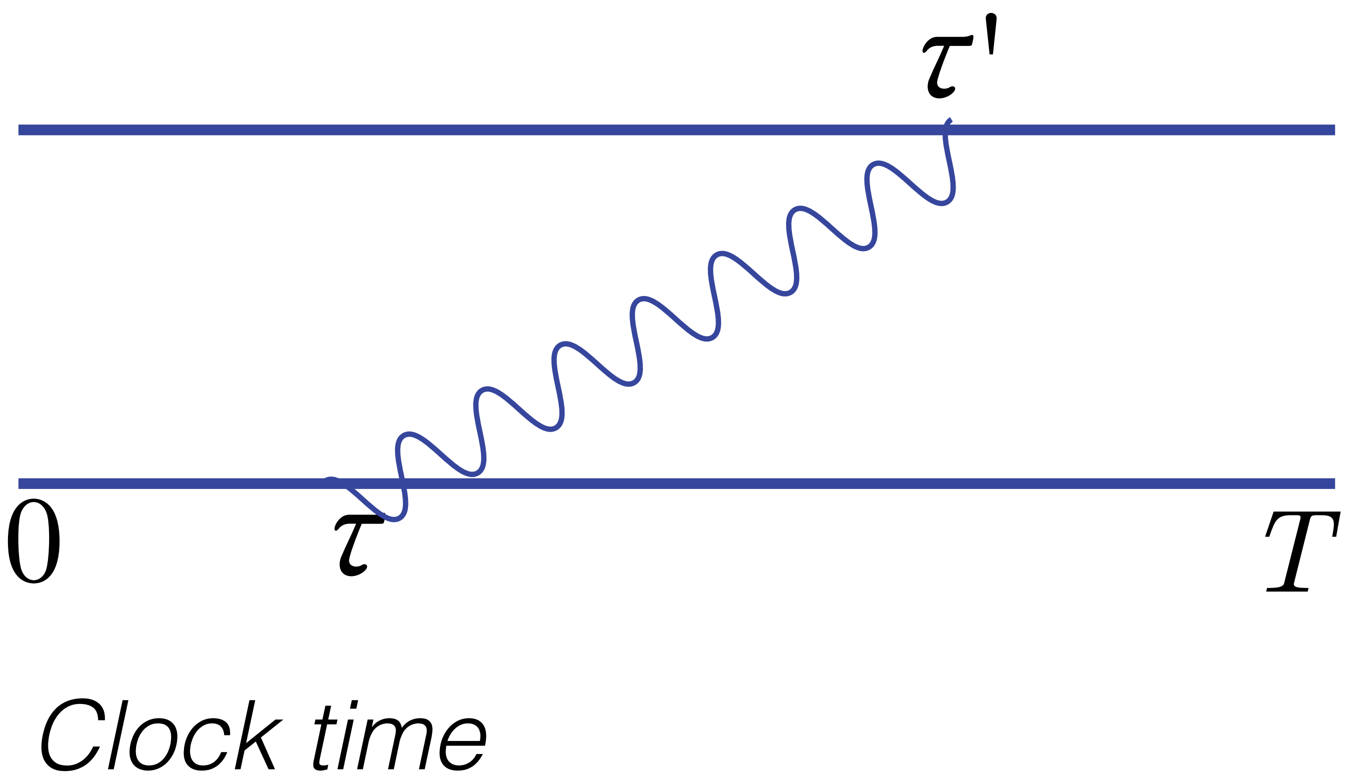
$$\exp(-\imath \kappa \tau)$$

$$\exp\left(\imath \left(\frac{w^2 - \kappa^2}{2w} - \kappa\right) \tau\right)$$

$$K(\alpha, \kappa) = \exp\left(-\imath \kappa \tau \left(1 - \frac{\alpha^2}{2}\right)\right)$$

- Scale out the 3 momentum
- Consider SQM propagator
- Consider combination of TQM and SQM proctors
- Expand in terms of scaling factor to 2nd power

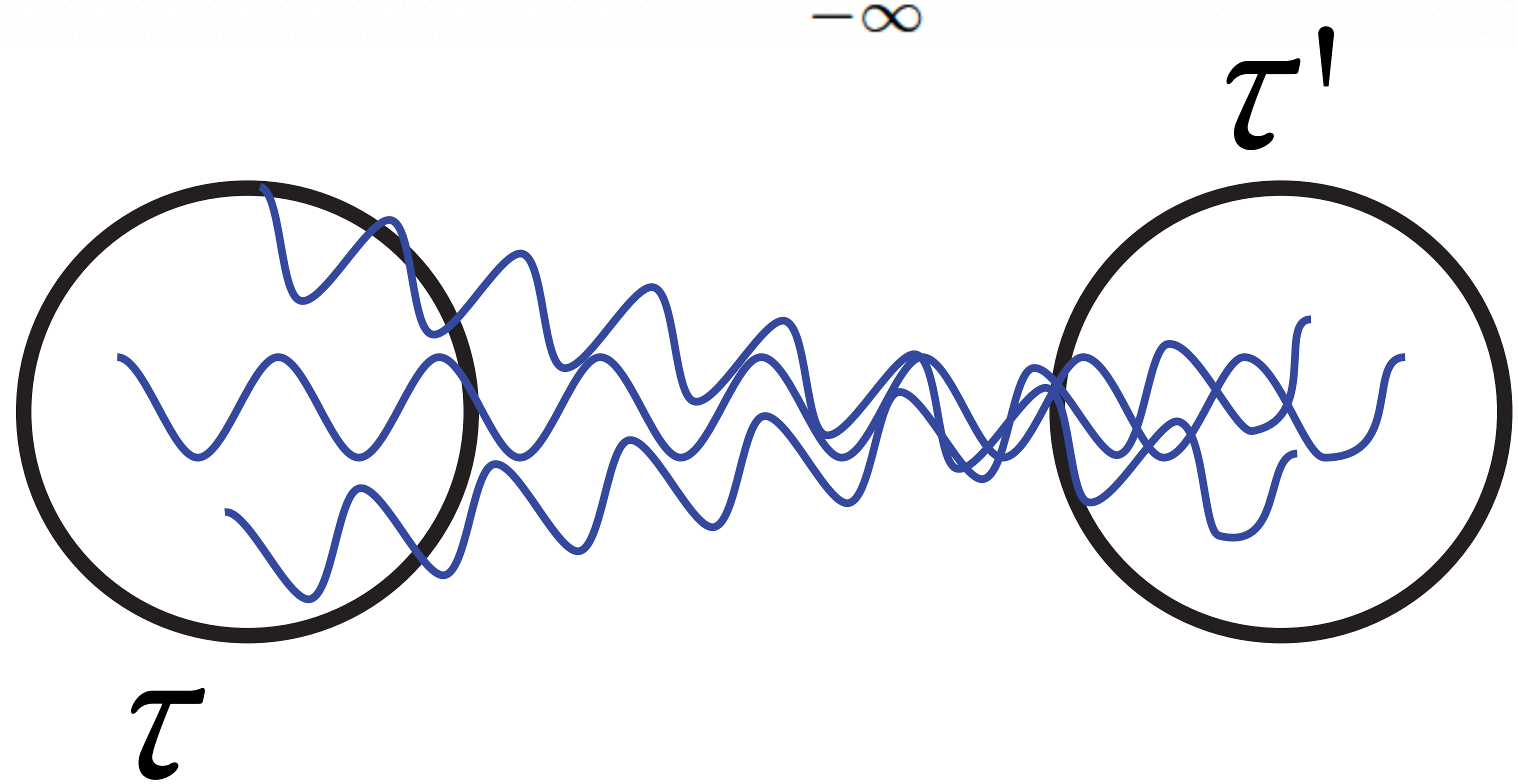
EFFECTIVE POTENTIAL



- Start with particle wave function
- Take kernel from start
- Integrate over clock time
- Basically variation on the retarded potential

$$\imath\frac{\partial}{\partial\tau}\varphi_\tau^A(x_1)=-\frac{p_A^2-m_A^2}{2m_A}\varphi_\tau^A(x_1)-e^2\int\limits_{-\infty}^{\tau}d\tau'\int dx_2K_{\tau\tau'}^{\gamma}(x_1-x_2)\,\varphi_\tau^A(x_2)$$

$$\imath\frac{\partial}{\partial\tau}\varphi_\tau^B(x_2)=-\frac{p_B^2-m_B^2}{2m_B}\varphi_\tau^B(x_2)-e^2\int\limits_{-\infty}^{\tau}d\tau'\int dx_1K_{\tau\tau'}^{\gamma}(x_2-x_1)\,\varphi_\tau^A(x_1)$$



$$x\equiv x_1-x_2$$

$$X\equiv\frac{m_Ax_1+m_Bx_2}{m_A+m_B}$$

$$\mu\equiv\frac{m_Am_B}{m_A+m_B};m_A\ll m_B\Rightarrow\mu\approx m_A$$

$$\imath\frac{\partial}{\partial\tau}\varphi_\tau(x)=-\frac{p^2-\mu^2}{2\mu}\varphi_\tau(x)-e^2\int\limits_{-\infty}^{\tau}d\tau'\int dx'K_{\tau\tau'}^{\gamma}(x-x')\,\varphi_\tau(x')$$

SEPARATION OF VARIABLES

$$\imath \frac{\partial}{\partial \tau} \psi_\tau = - \frac{(E - \Phi)^2 - \left(\vec{p} - \vec{A}\right)^2 - \mu^2}{2\mu} \psi_\tau$$

$$\imath \frac{\partial}{\partial \tau} \psi_\tau = - \frac{E^2}{2\mu} \psi_\tau + \left(\frac{\vec{p}^2}{2\mu} + \Phi \right) \psi_\tau - \frac{(E - \mu)\,\Phi}{\mu} \psi_\tau$$

$$\phi_w(t) = \frac{1}{\sqrt{2\pi}} \exp(-\imath w t)$$

$$\psi(t, \vec{r}) = \sum_{wnlm} c_{wnlm} \phi_w(t) \psi_{nlm}(\vec{r})$$

- Write out using minimal substitution
- Drop high order terms
- Separates
- Start of perturbation expansion

Rules of interactions

$$\psi_{nml}(t,\vec{r}) = \varphi_{nlm}(t)\psi_{nlm}^{(SQM)}(\vec{r})$$

$$\varphi_{nlm}(t) = \sqrt[4]{\frac{1}{\pi\sigma_t^2}} e^{-\imath\bar{E}(t-\tau)-\frac{(t-\tau)^2}{2\sigma_{nlm}^2}}$$

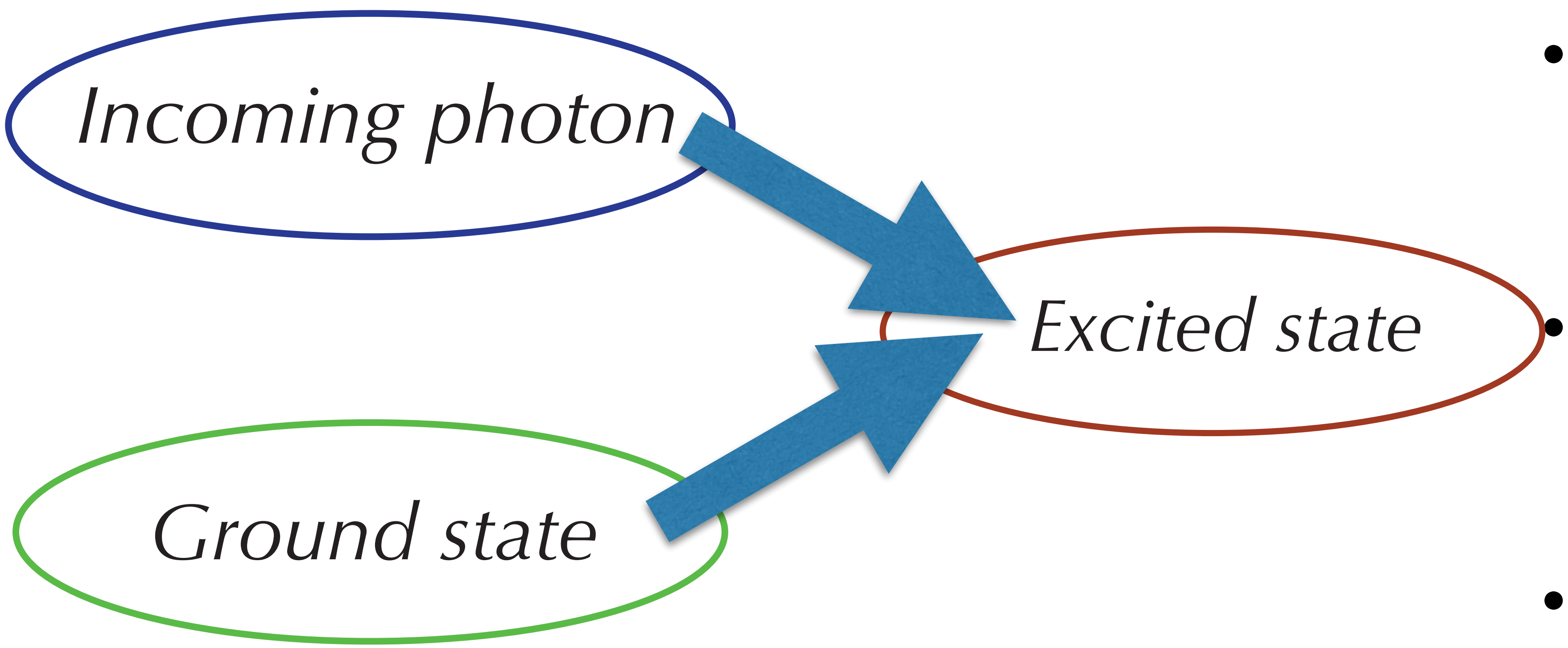
$$\sigma_{nlm}^2 = \frac{1}{\hat{\sigma}_E^2}$$

$$\hat{\sigma}_E^2 = \frac{\Delta E_{nlm}}{2}$$

- Use simpler direct product form of the wave function
- Same formalism as SQM otherwise
- Smoother transitions
- And generally more complex dynamics

Absorption

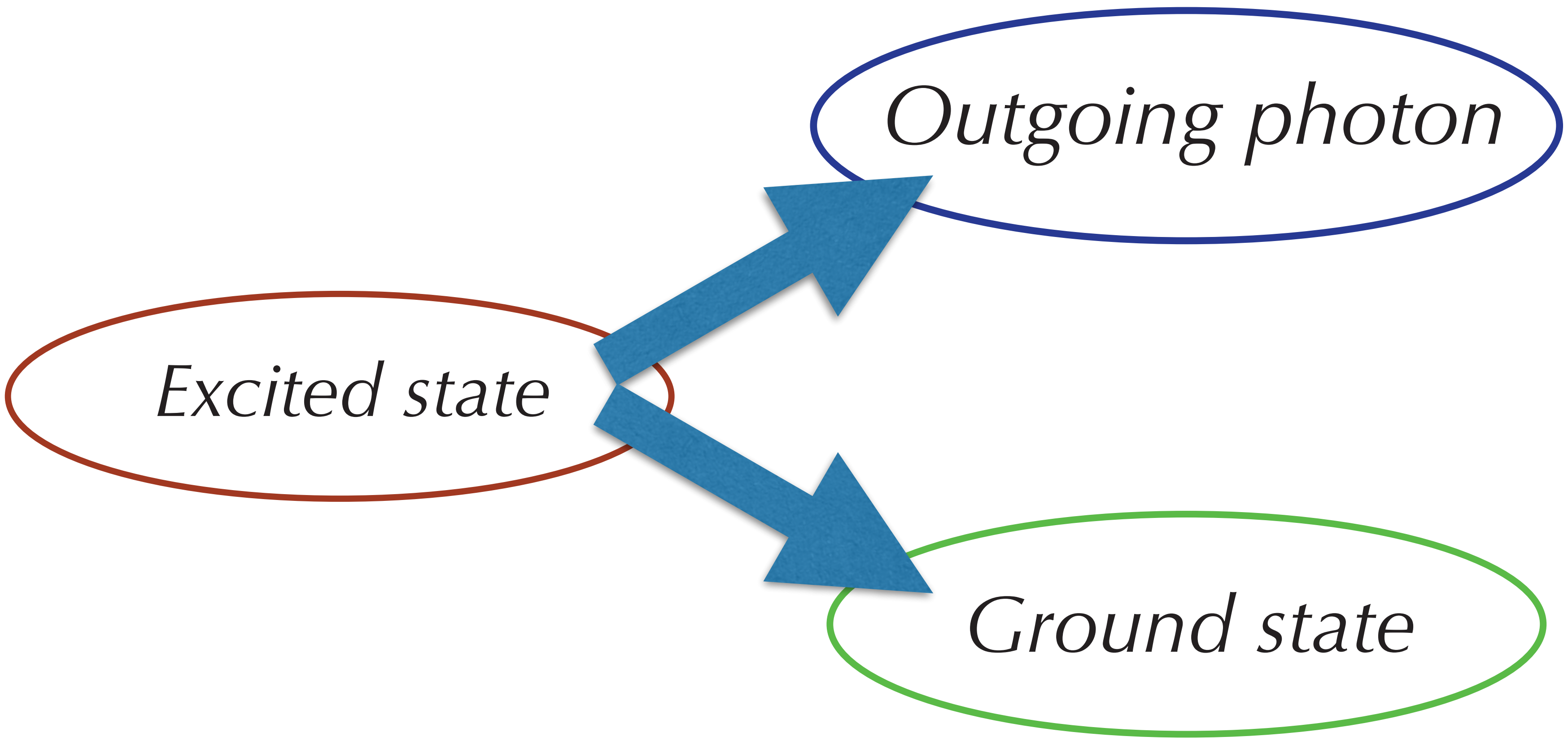
$$\langle \psi_1 | \vec{A} \cdot \vec{p} | \psi_0 A \rangle$$



- 3d to 4d
- Hysteresis
- Longer times, otherwise not usually much change
- Is a model for measurement
- No collapse, smooth transition via time

Emission

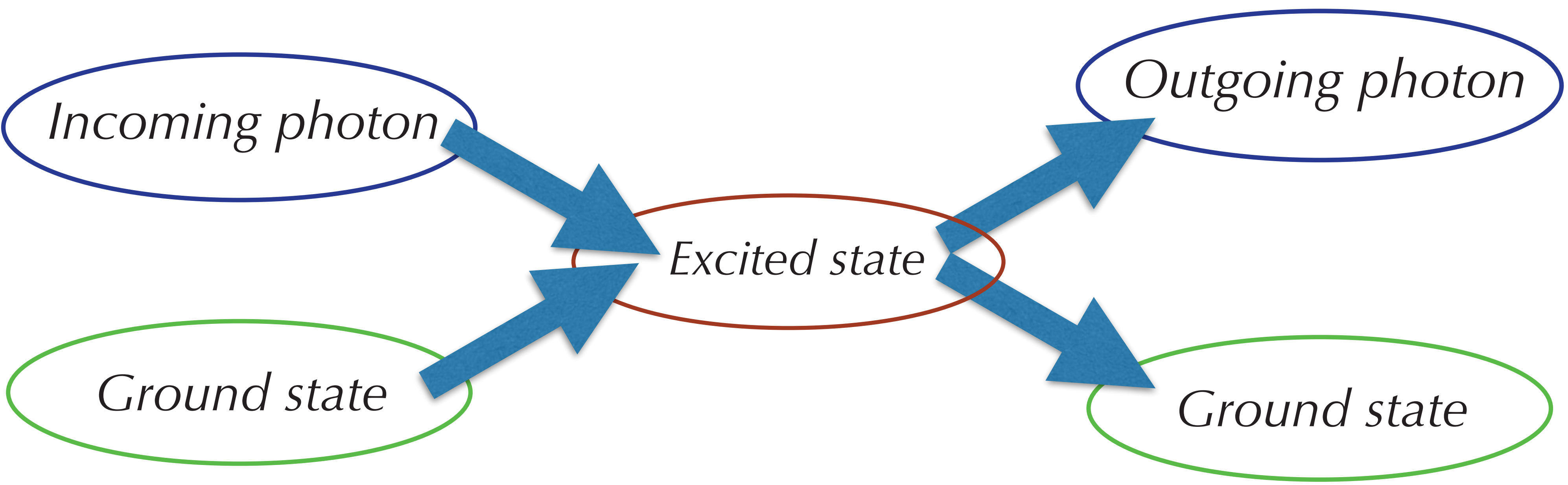
$$\langle \psi_0 A | \vec{A} \cdot \vec{p} | \psi_1 \rangle$$



- 3D to 4D
- Rate
- Need to specify both the 3 momenta and the time of arrival
- No jump
- Smooth transition via time

Scattering

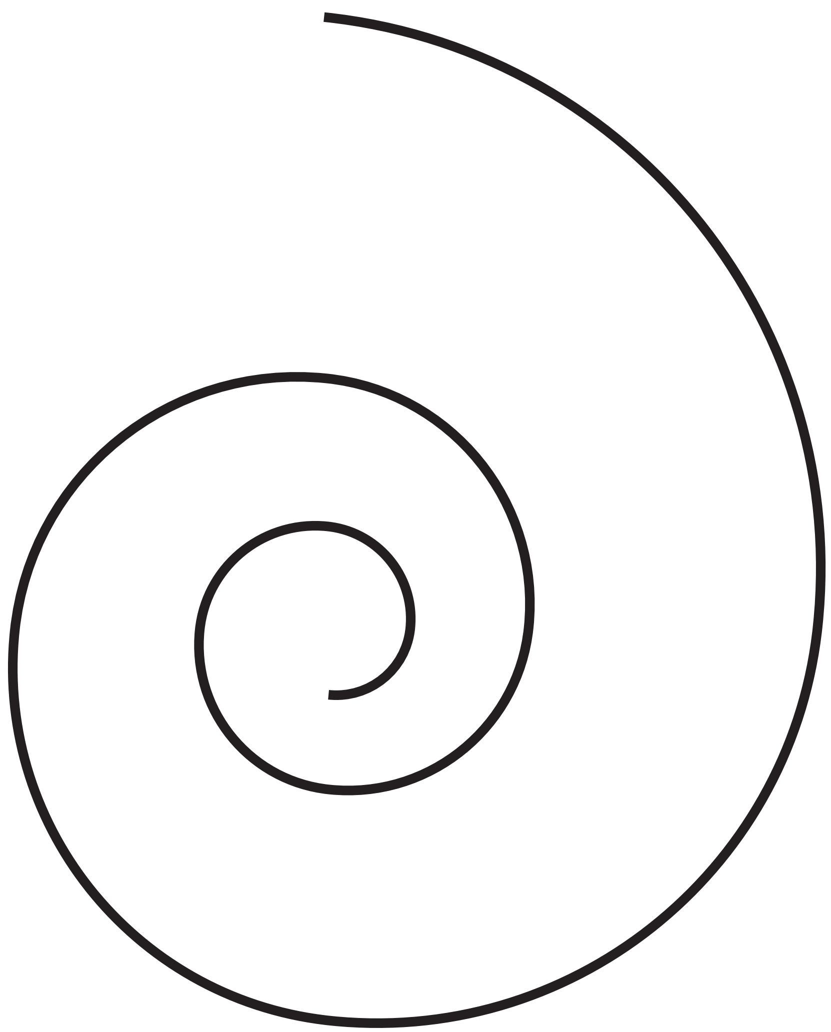
$$\langle \psi_1 | \vec{A} \cdot \vec{p} \, | \psi_0 A \rangle \; \langle \psi_0 A | \vec{A} \cdot \vec{p} \, | \psi_1 \rangle$$



- 3D to 4D
- Decoherence effects
- Some possibility of "ringing" effects, for shorter times
- No pure intermediate states
- Smooth connection via time
- Is a model for decoherence

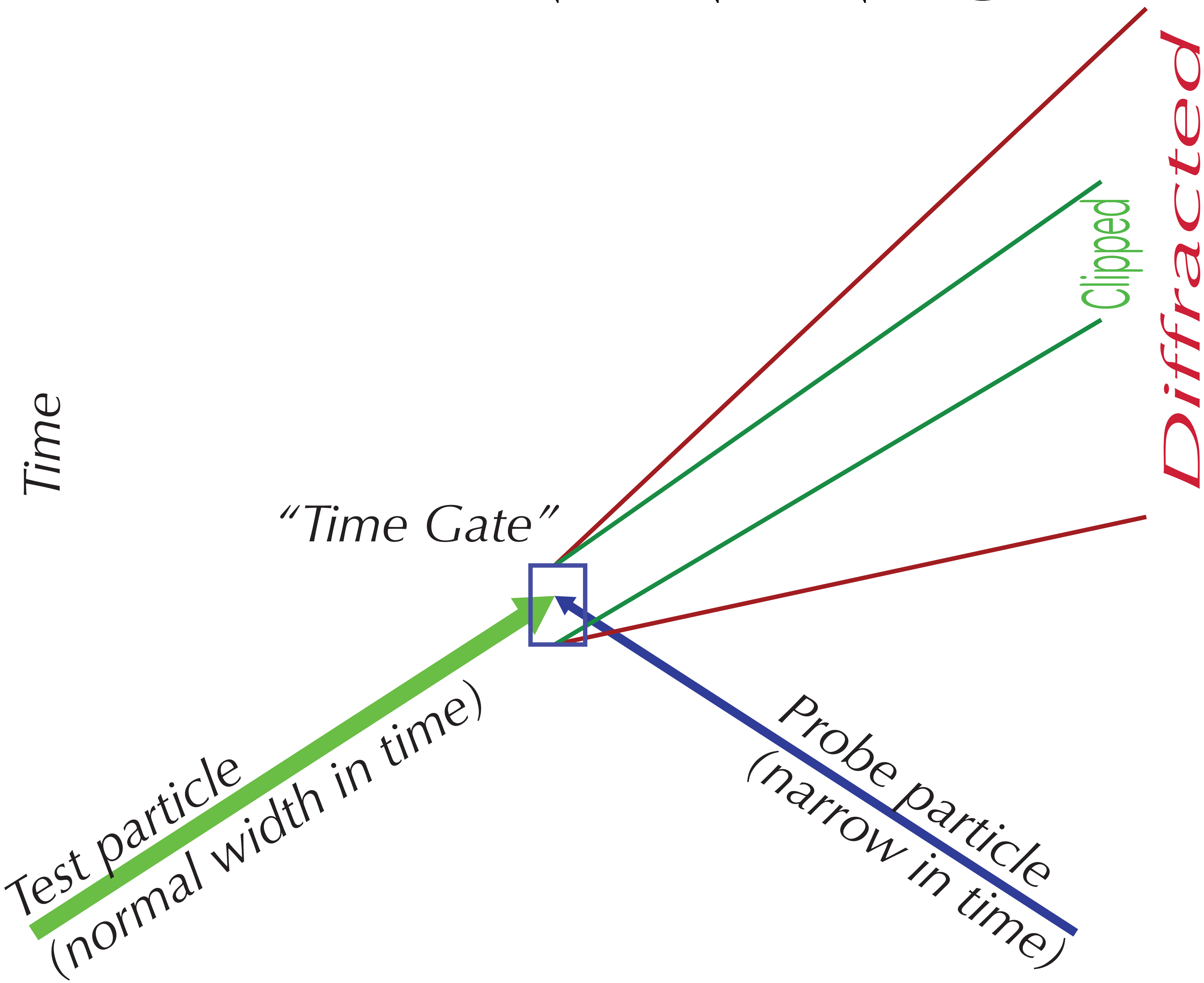
Implications for measurement

$$\tau_{fall} = 1.6\,10^{-11} s$$

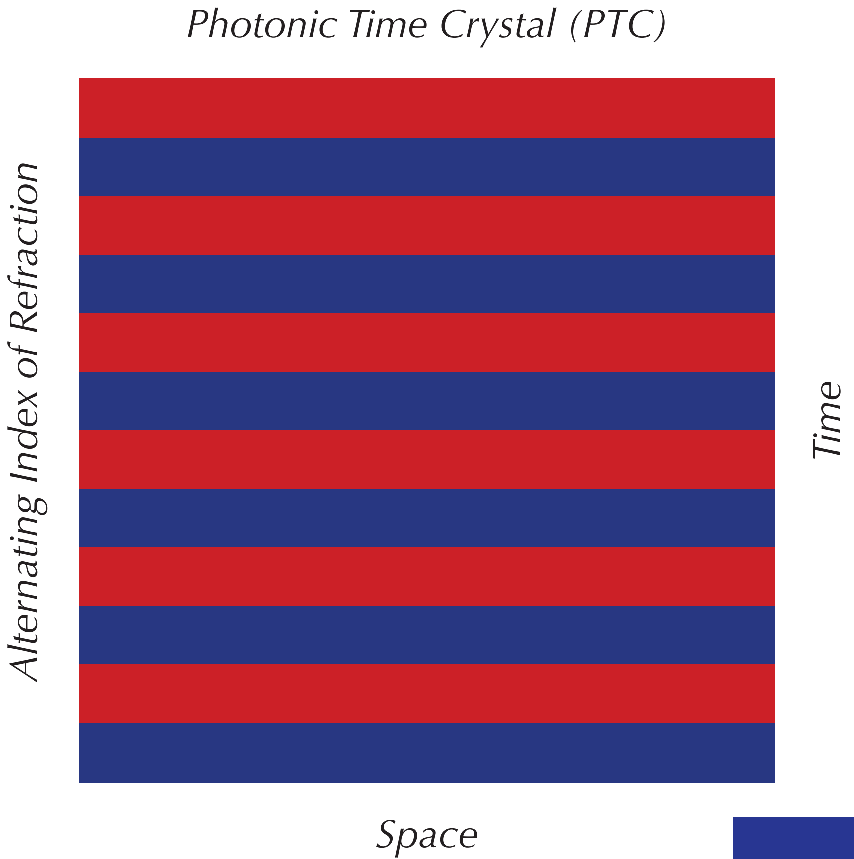


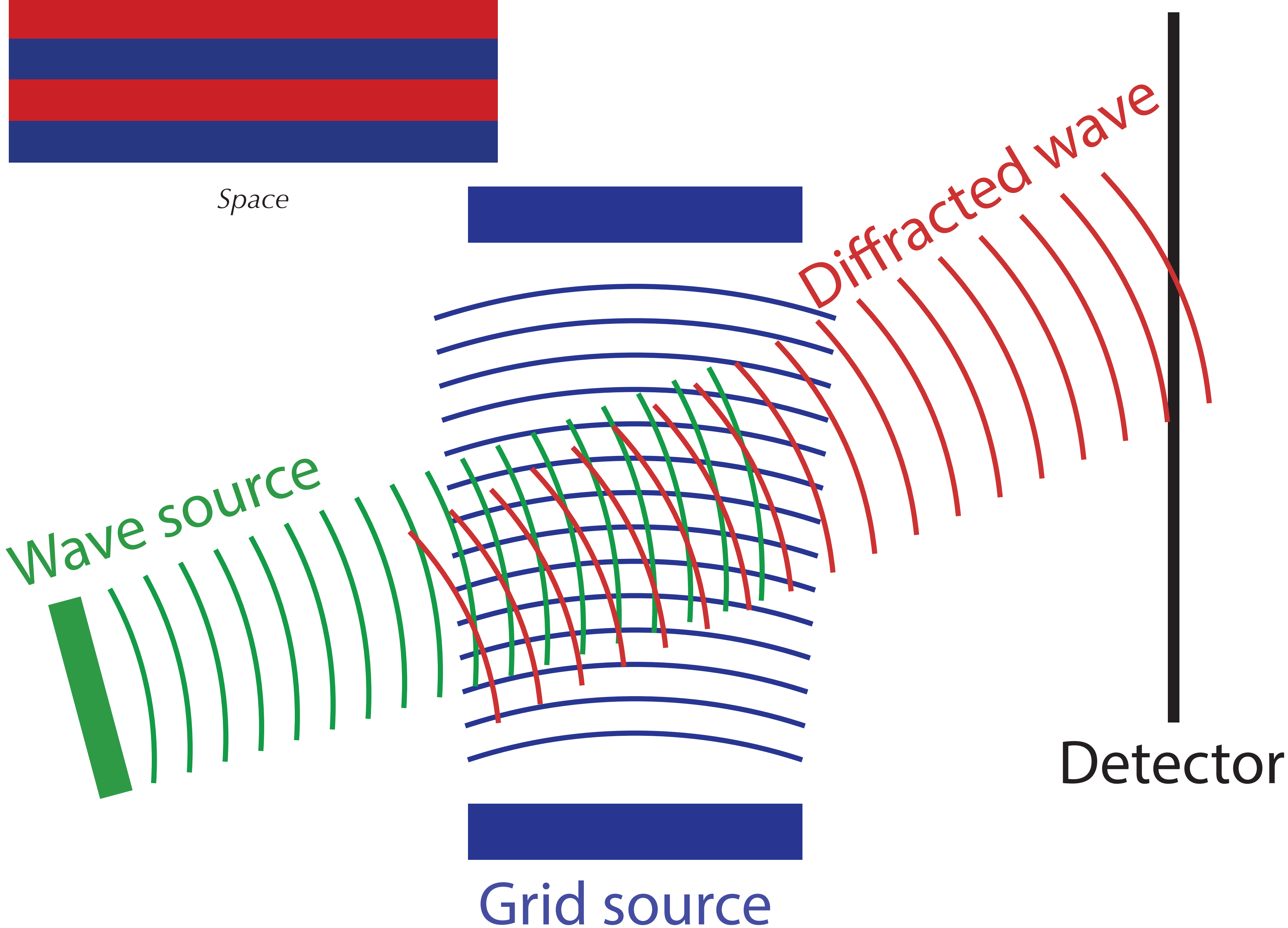
- ε/δ approach
- Absorption/Problem of measurement
- Emission/Schrödinger's cat
- Scattering/Decoherence
- One reality/two languages

Heisenberg uncertainty in Time/Energy



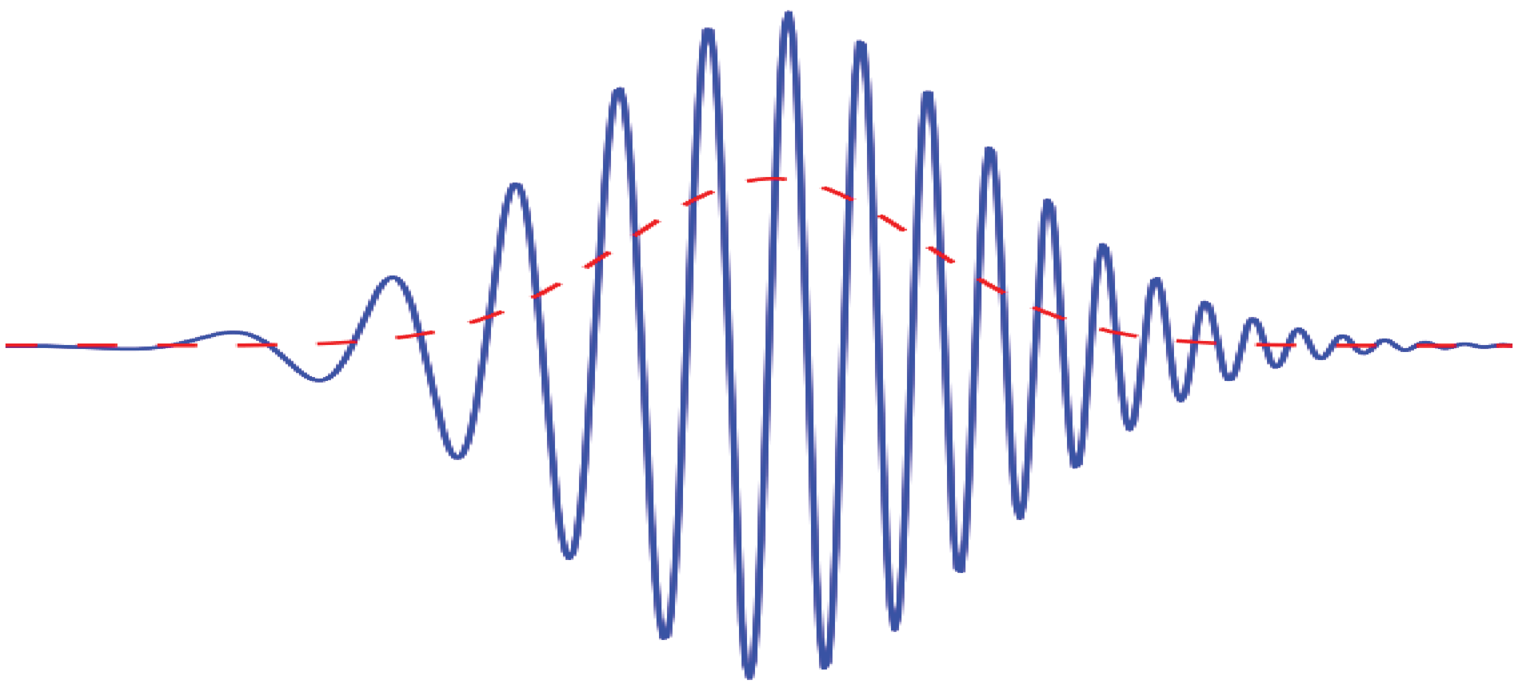
Use of time crystals to detect dispersion in time





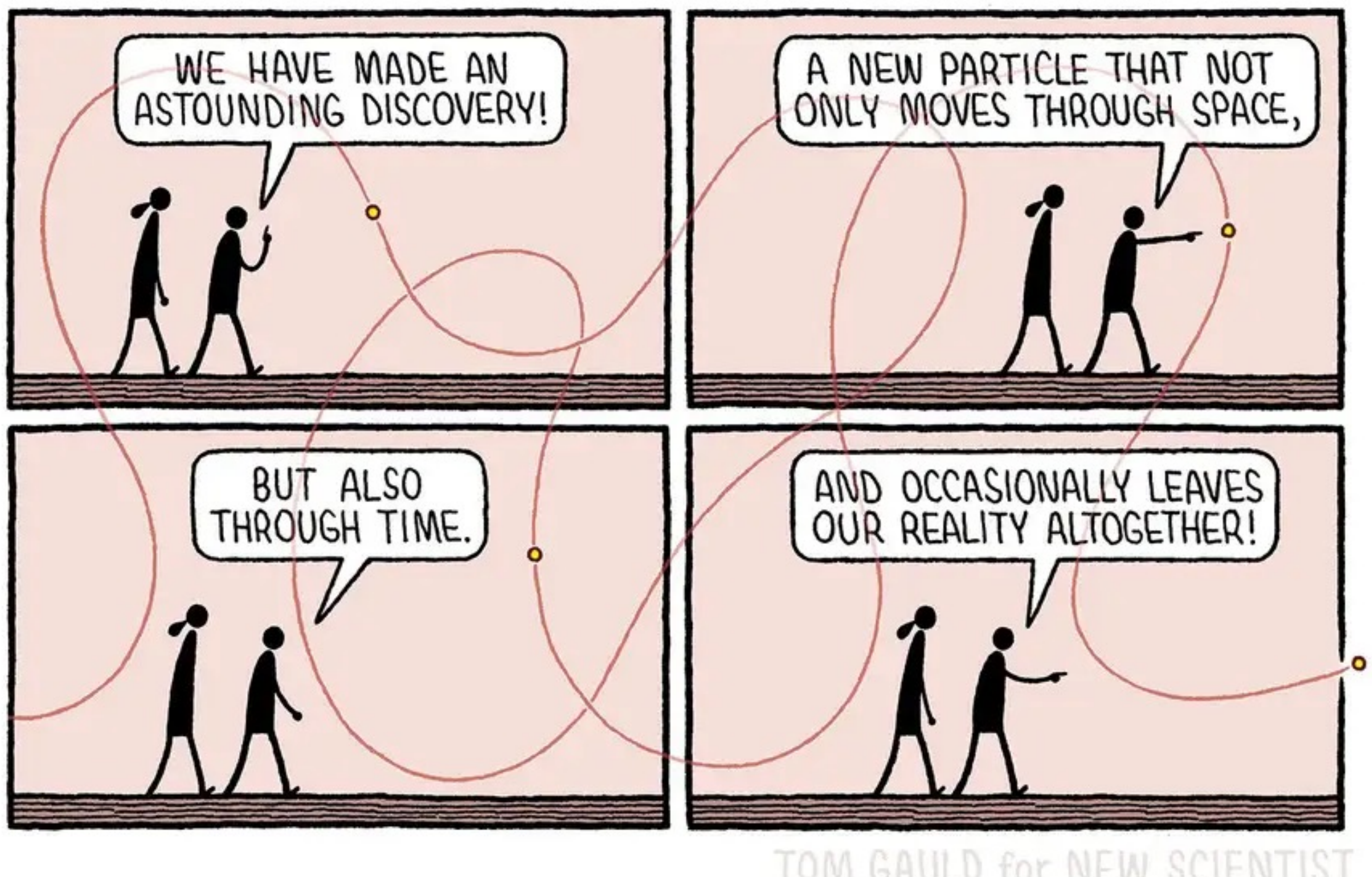
- Wave functions extend in time
- therefore can be diffracted by time crystals
- Multiple cycles
- High frequency

"Chirps"

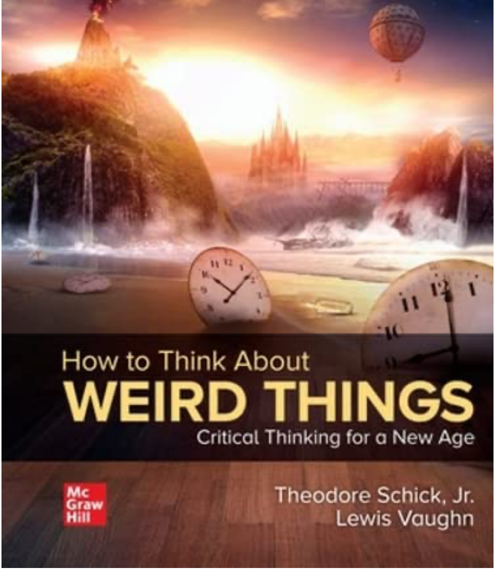


- Short times
- High frequencies

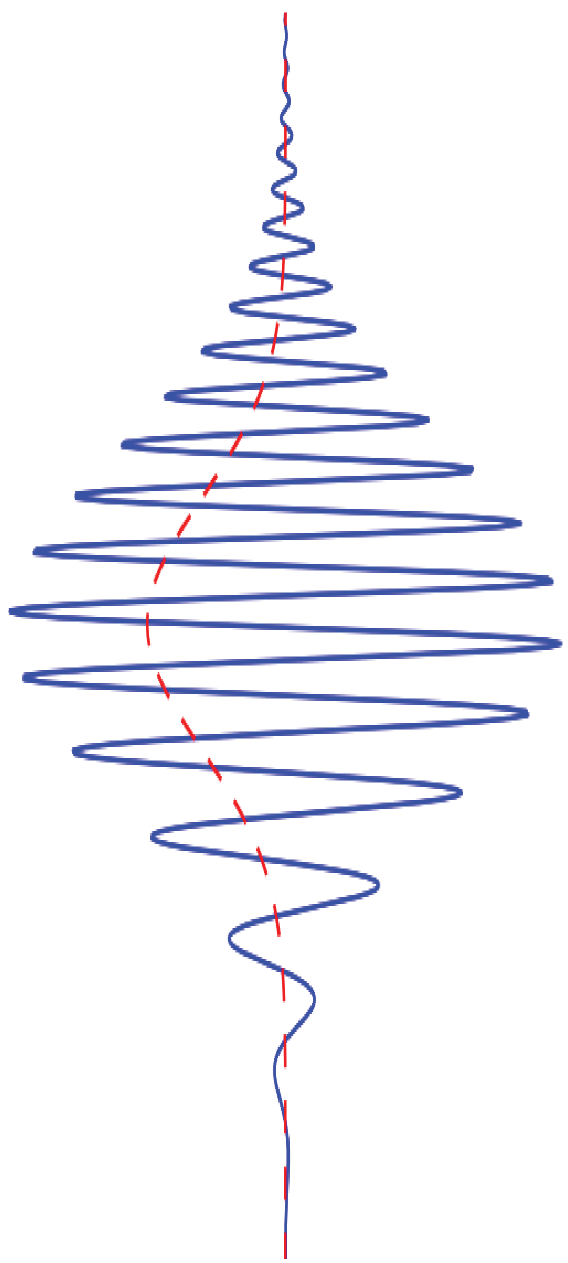
Internal State / Hysteresis



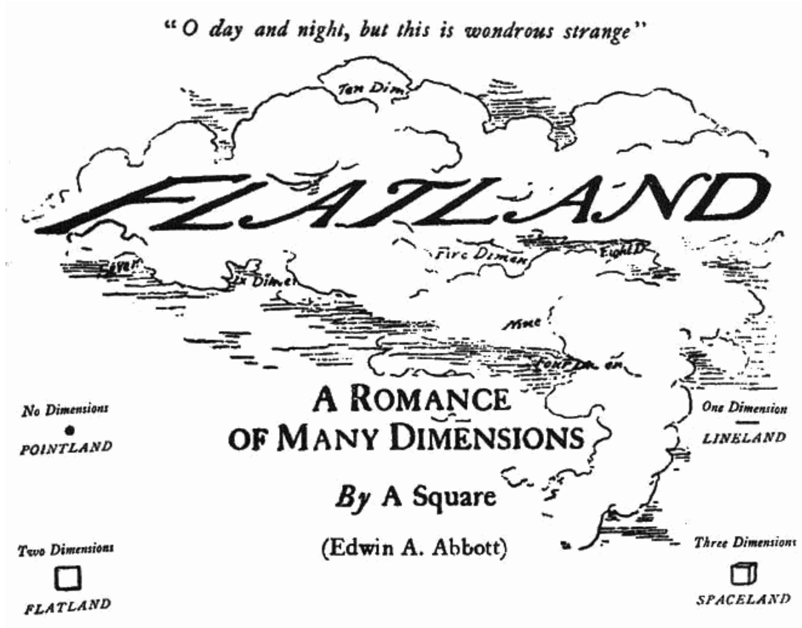
- Internal variable
- Not a hidden variable, fully quantum
- Smoothes over the jumps
- Provides a secret memory
- Averages out from decoherence



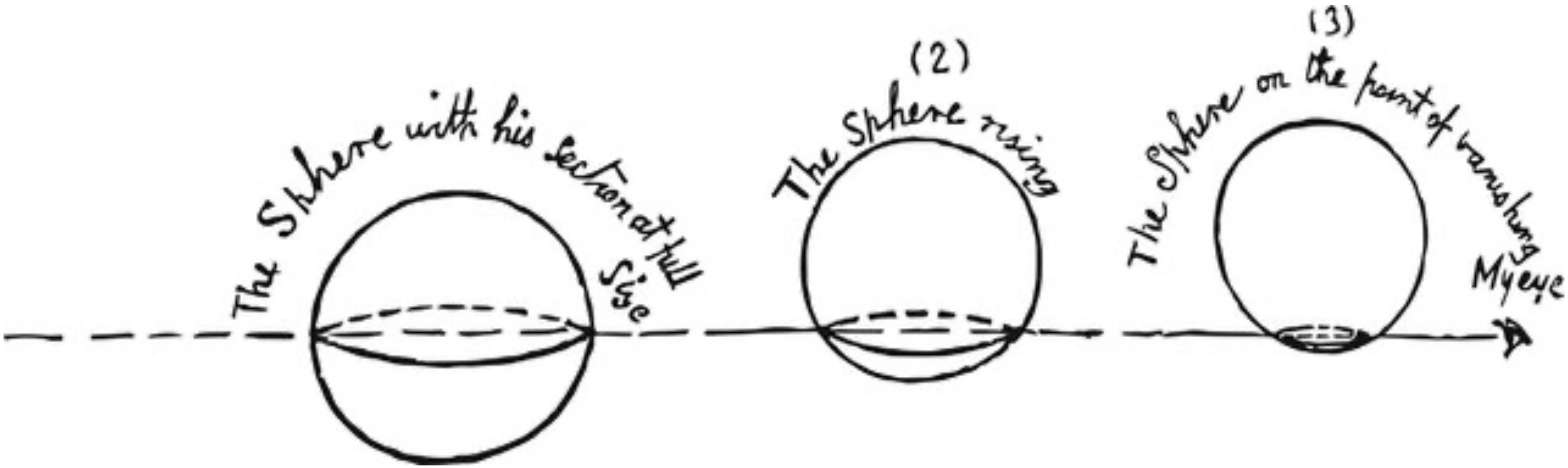
How to think about weird things – Schick & Vaughan



- Testable: large variety of tests
- Scope: short times, high energies, gravitons
- Simple: conceptual simpler, calculation more complex
- Consistent: with what is known, self-consistent
- Fruitful: new lines of attack



Questions are more important than answers – Einstein



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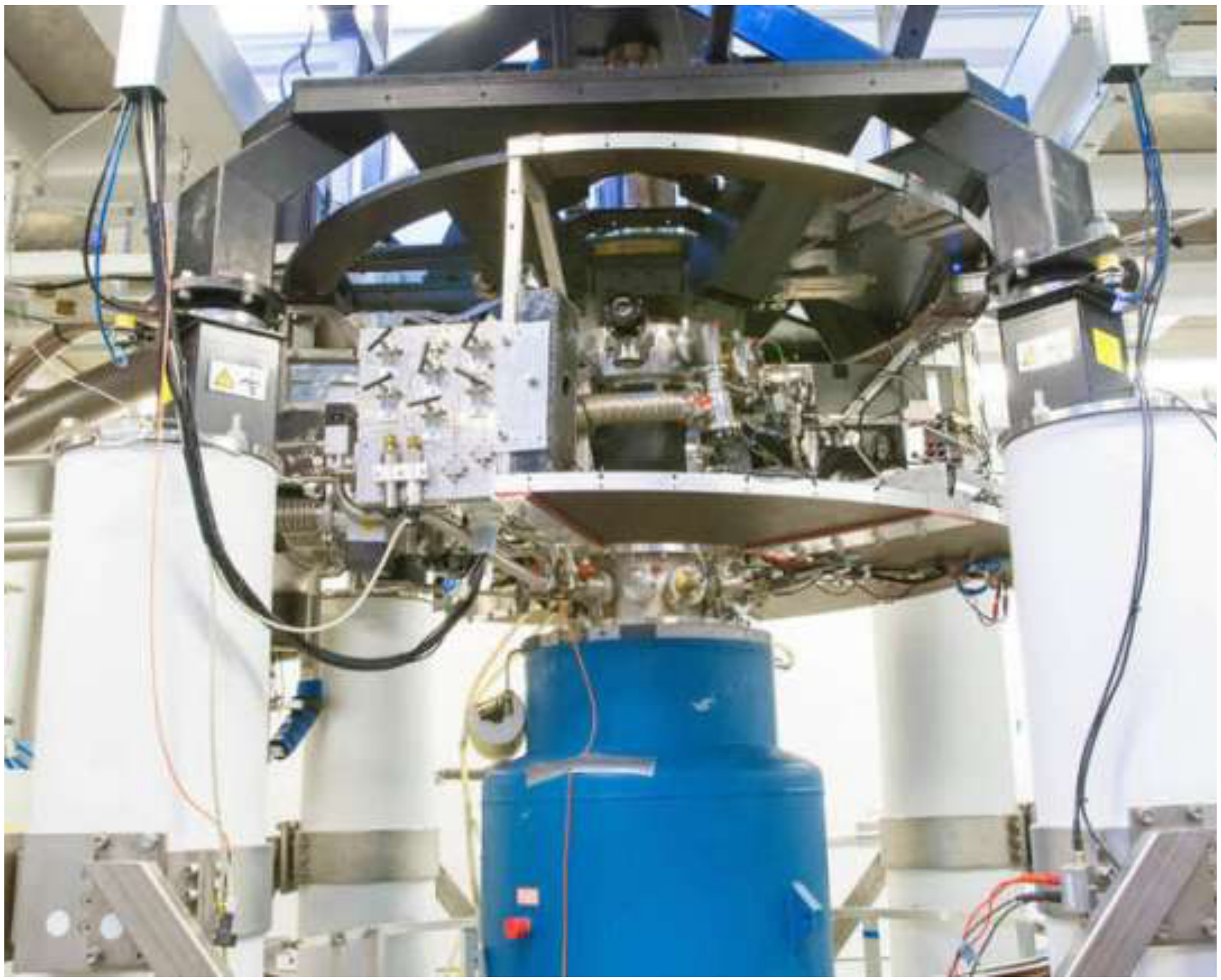
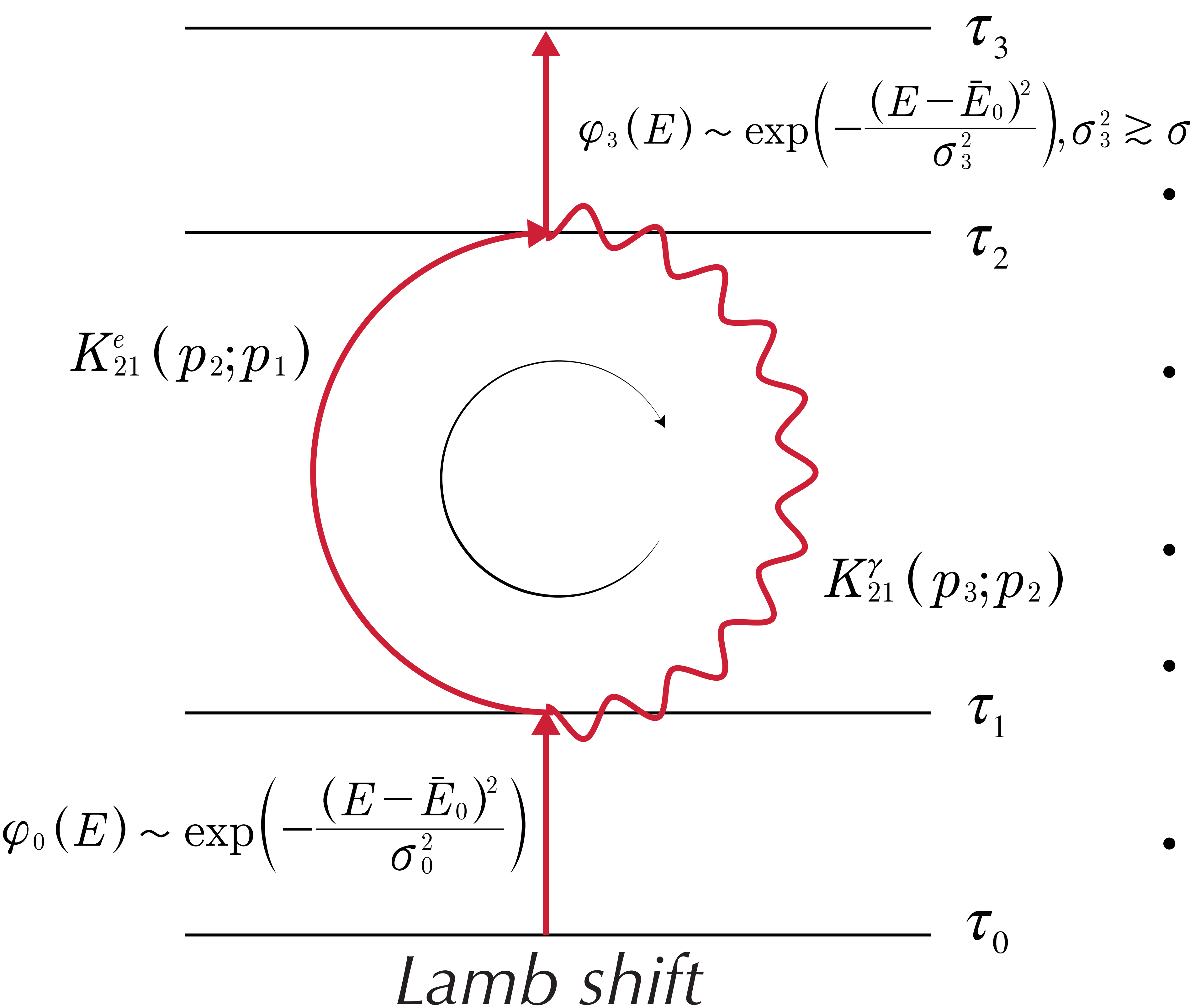


- IARD
- Aalto university
- Martin Land
- Ferne Cohen Welch
- Johnathan Smith

Lamb Shift



- Start and finish are bound states
- Fixed clock time, first integral, second integral
- Acts as mass term
- But with additional dispersion
- Dispersion in time AND entanglement in time regularize the integrals

First Few Atomic Wave Functions

$$\psi_{10}(p) = \sqrt{\frac{32}{\pi}} \frac{1}{(p^2+1)^2}$$

$$\psi_{20}(p) = \sqrt{\frac{4}{\pi}} \frac{1-4p^2}{\left(p^2+\frac{1}{4}\right)^2 (4p^2+1)}$$

$$\psi_{21}(p) = \sqrt{\frac{4}{\pi}} \frac{p}{\sqrt{3}\left(p^2+\frac{1}{4}\right)^3}$$

- Simple forms
- p is momentum times Bohr radius
- Normalized with respect to p
- Independent of m