Relativistic Transformations of Thermodynamics, Relativistic Statistical Mechanics and Einstein's Dual Theory

Dr. Gonzalo Ares de Parga.[1].
Dr. Tepper L. Gill[2].

[1] Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Ciudad de México, C. P. 07738, México, E-mail: garesdepargaa@ipn.mx
[2] Department of Mathematics, Physics and E$\backslash\&CE, Howard University$, Washington DC 20059 USA, E-mail: tgill@howard.edu

Abstract

Abstract

A brief description of the Redefined Relativistic Thermodynamics is exposed relating it with the Relativistic Statistical Mechanics and showing that Einstein-Planck, Ott and Rohrlich proposals represent particular choices of a reference frame where the instantaneity is considered.

The Einstein's dual theory is described arriving to the conclusion that for a system of particles a universal time exists called the proper time.

The instantaneity can be considered in the frame where the observer is at rest in the canonical dual Hamiltonian center of mass.

This will relate the different proposals to the Proper Time of the system.

Keywords: Redefined Relativistic Thermodynamics; relativistic transformation of the temperature; proper time

Abstract

Abstract

A brief description of the Redefined Relativistic Thermodynamics is exposed relating it with the Relativistic Statistical Mechanics and showing that Einstein-Planck, Ott and Rohrlich proposals represent particular choices of a reference frame where the instantaneity is considered.
The Einstein's dual theory is described arriving to the conclusion that for a system of particles a universal time exists called the proper time.
The instantaneity can be considered in the frame where the observer is at rest in the canonical dual Hamiltonian center of mass.
This will relate the different proposals to the Proper Time of the system.

Keywords: Redefined Relativistic Thermodynamics; relativistic transformation of the temperature; proper time

Abstract

A brief description of the Redefined Relativistic Thermodynamics is exposed relating it with the Relativistic Statistical Mechanics and showing that Einstein-Planck, Ott and Rohrlich proposals represent particular choices of a reference frame where the instantaneity is considered.
The Einstein's dual theory is described arriving to the conclusion that for a system of particles a universal time exists called the proper time.
The instantaneity can be considered in the frame where the observer is at rest in the canonical dual Hamiltonian center of mass.
This will relate the different proposals to the Proper Time of the system.

Keywords: Redefined Relativistic Thermodynamics; relativistic transformation of the temperature; proper time

Abstract

A brief description of the Redefined Relativistic Thermodynamics is exposed relating it with the Relativistic Statistical Mechanics and showing that Einstein-Planck, Ott and Rohrlich proposals represent particular choices of a reference frame where the instantaneity is considered.
The Einstein's dual theory is described arriving to the conclusion that for a system of particles a universal time exists called the proper time.
The instantaneity can be considered in the frame where the observer is at rest in the canonical dual Hamiltonian center of mass.
This will relate the different proposals to the Proper Time of the system.

Keywords: Redefined Relativistic Thermodynamics; relativistic transformation of the temperature; proper time

Abstract

A brief description of the Redefined Relativistic Thermodynamics is exposed relating it with the Relativistic Statistical Mechanics and showing that Einstein-Planck, Ott and Rohrlich proposals represent particular choices of a reference frame where the instantaneity is considered.
The Einstein's dual theory is described arriving to the conclusion that for a system of particles a universal time exists called the proper time.
The instantaneity can be considered in the frame where the observer is at rest in the canonical dual Hamiltonian center of mass.
This will relate the different proposals to the Proper Time of the system.

Keywords: Redefined Relativistic Thermodynamics; relativistic transformation of the temperature; proper time

1- Introduction

In 1907, the laws of relativistic transformations of thermodynamics were proposed by Planck [1] and Einstein [2] (PE). There were no important discrepancies to the respect and all the theory was resumed in the books of Tolman [3] and Pauli [4].

However, at the end of his life Einstein communicated to van Laue [5] his doubts about the validity of the theory.

From it, in 1964, Ott [6] published his posthumous paper where he proposed a new set of relativistic transformation laws in thermodynamics and the subject newly called the attention of many researches (Ott proposal (O)).

Consequently, countless number of different proposals were published [7]. More than a hundred of articles appeared during the sixties and the seventies of the past century defending or contradicting the PE proposal.

1- Introduction

In 1907, the laws of relativistic transformations of thermodynamics were proposed by Planck [1] and Einstein [2] (PE). There were no important discrepancies to the respect and all the theory was resumed in the books of Tolman [3] and Pauli [4].
However, at the end of his life Einstein communicated to van Laue [5] his doubts about the validity of the theory.
From it, in 1964, Ott [6] published his posthumous paper where he proposed a new set of relativistic transformation laws in thermodynamics and the subject newly called the attention of many researches (Ott proposal (O)).
Consequently, countless number of different proposals were published [7]. More than a hundred of articles appeared during the sixties and the seventies of the past century defending or contradicting the PE proposal.

1- Introduction

In 1907, the laws of relativistic transformations of thermodynamics were proposed by Planck [1] and Einstein [2] (PE). There were no important discrepancies to the respect and all the theory was resumed in the books of Tolman [3] and Pauli [4].
However, at the end of his life Einstein communicated to van Laue [5] his doubts about the validity of the theory.
From it, in 1964, Ott [6] published his posthumous paper where he proposed a new set of relativistic transformation laws in thermodynamics and the subject newly called the attention of many researches (Ott proposal (O)).
Consequently, countless number of different proposals were published [7]. More than a hundred of articles appeared during the sixties and the seventies of the past century defending or contradicting the PE proposal.

1- Introduction

In 1907, the laws of relativistic transformations of thermodynamics were proposed by Planck [1] and Einstein [2] (PE). There were no important discrepancies to the respect and all the theory was resumed in the books of Tolman [3] and Pauli [4].
However, at the end of his life Einstein communicated to van Laue [5] his doubts about the validity of the theory.
From it, in 1964, Ott [6] published his posthumous paper where he proposed a new set of relativistic transformation laws in thermodynamics and the subject newly called the attention of many researches (Ott proposal (O)).
Consequently, countless number of different proposals were published [7]. More than a hundred of articles appeared during the sixties and the seventies of the past century defending or contradicting the PE proposal.

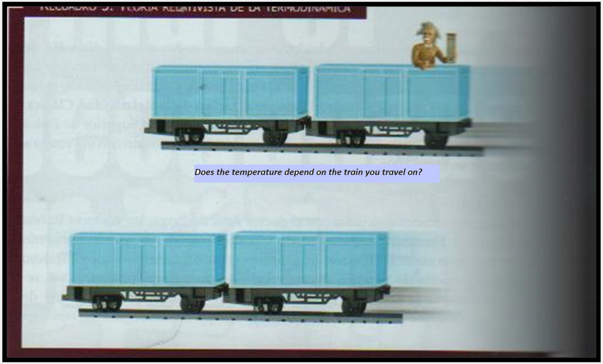

Figure 1: The temperature depends on the train you travel

Many theories were presented incorporating new variants. For example, we can mention the works realized by Arzeliès [8], Rohrlich [9] and Landsberg [10] (Landsberg proposal (L)) among others.

Landsberg [11] suggested that just the experiment could clarify the controversies. Nevertheless, in 1968, Balescu [7] presented a statistical theory of the PE proposal demonstrating that not only it was the unique model which preserves the invariant form of thermodynamics but any other scheme could be obtained by using a gauge transformation.

However, at the beginning of this century, most of the papers were close to the PE theory as it is the case of Sieniutycz's article [12].

Many theories were presented incorporating new variants. For example, we can mention the works realized by Arzeliès [8], Rohrlich [9] and Landsberg [10] (Landsberg proposal (L)) among others.

Landsberg [11] suggested that just the experiment could clarify the controversies. Nevertheless, in 1968, Balescu [7] presented a statistical theory of the PE proposal demonstrating that not only it was the unique model which preserves the invariant form of thermodynamics but any other scheme could be obtained by using a gauge transformation.

However, at the beginning of this century, most of the papers were close to the PE theory as it is the case of Sieniutycz's article [12].

Many theories were presented incorporating new variants. For example, we can mention the works realized by Arzeliès [8], Rohrlich [9] and Landsberg [10] (Landsberg proposal (L)) among others.

Landsberg [11] suggested that just the experiment could clarify the controversies. Nevertheless, in 1968, Balescu [7] presented a statistical theory of the PE proposal demonstrating that not only it was the unique model which preserves the invariant form of thermodynamics but any other scheme could be obtained by using a gauge transformation.

However, at the beginning of this century, most of the papers were close to the PE theory as it is the case of Sieniutycz's article [12].

Classification of the different proposals done by Balescu

Coshs=γ; tanhs=u/c		V	P	S	T	dQ	E	F
I	Planck, Einstein, Hasenöhrl, Jüttner, Textos, Pathria, Guessous, de Broglie, Hillion, Staruszkiewicz, Penney, Eberly-Kujawski	$V = \frac{V_o}{\cosh s}$	$p = p_o$	$S = S_o$	$T = \frac{T_o}{\cosh s}$	$\delta Q = \frac{\delta Q_o}{\cosh s}$	$E = E_o \cosh s + p_o V_o \sinh s \tanh s$	$F = \frac{F_o}{\cosh s}$
II	Kibble, Møller				$T = T_o \cosh s$	$\delta Q = \delta Q_o \cosh s$		
	Brevik						Complicado	No dice
	Ott, Arseliéz, Gamba, Børs, Souriau						$E = E_o \cosh s$	$F = F_o \cosh s$
	Sutcliffe		$p_m = p_o$, $p_{th} = p_o \cosh^2 s$					
I-II	Rohrlich	$V = V_o / \cosh s$, $V' = V_o \cosh s$	$p = p_o$		$T = T_o / \cosh s$, $T' = T_o \cosh s$	No dice	$E = E_o$	No dice
III	Landsberg	$V = \frac{V_o}{\cosh s}$			$T = T_o$	$\delta Q = \delta Q_o$	$E = E_o / \cosh s$, $E' = E_o \cosh s$	$F = F_o$
	Landsberg-Johns					$\delta Q = \delta Q_o / \cosh s$	$E = E_o \cosh s + p_o V_o \sinh s \tanh s$	

Figure 2: Balescu's classification of the different proposal about relativistic transformation in Thermodynamics

In the meantime, Landsberg and Matsas [13], [14] published two works where they claimed having demonstrated that due to the equivalence between the frequency number density for a moving black-body and the excitation rate of the Unruh-DeWitt detector [15], a relativistic transformation law of the temperature cannot be defined. Their reasoning was based on the non-Planckian form of the frequency number density. Apparently, the Landsberg and Matsas papers explained the origin of all controversies.

However, Ares de Parga et al [16] (the AA proposal) integrated the frequency density number over all the particle frequencies and obtained the real number of particles. Then they calculated the internal energy multiplying the frequency number density by the frequency quantum of energy and integrating over all the frequencies, they got the PE transformation law of the energy.

In the meantime, Landsberg and Matsas [13], [14] published two works where they claimed having demonstrated that due to the equivalence between the frequency number density for a moving black-body and the excitation rate of the Unruh-DeWitt detector [15], a relativistic transformation law of the temperature cannot be defined. Their reasoning was based on the non-Planckian form of the frequency number density. Apparently, the Landsberg and Matsas papers explained the origin of all controversies.
However, Ares de Parga et al [16] (the AA proposal) integrated the frequency density number over all the particle frequencies and obtained the real number of particles. Then they calculated the internal energy multiplying the frequency number density by the frequency quantum of energy and integrating over all the frequencies, they got the PE transformation law of the energy.

Using finite time thermodynamics, Ares de Parga et al [16] found an exact differential which permitted them to define thermal quantities which preserve the invariant form of thermodynamics that was misconceived in the PE and Balescu proposals. The AA proposal was based on comparing the transformation law of the free Helmholtz energy [17] with the transformation law of the internal energy. The final conclusion consists in accepting that the redefined relativistic thermodynamics permits not only a well definition of the temperature and its relativistic transformation but also an invariant form of the Thermodynamics.

In this order of ideas, we can assure the existence of a relativistic temperature, which transforms as: $T = \gamma^{-1} T_0$, where T, T_0 are the temperatures at the moving system and at the rest frame respectively and $\gamma = 1/\sqrt{1 - u^2/c^2}$, where u represents the velocity between both systems. With this result, we are able to analyze the black-body distribution of $2.7K$ radiation background.

Using finite time thermodynamics, Ares de Parga et al [16] found an exact differential which permitted them to define thermal quantities which preserve the invariant form of thermodynamics that was misconceived in the PE and Balescu proposals. The AA proposal was based on comparing the transformation law of the free Helmholtz energy [17] with the transformation law of the internal energy. The final conclusion consists in accepting that the redefined relativistic thermodynamics permits not only a well definition of the temperature and its relativistic transformation but also an invariant form of the Thermodynamics.

In this order of ideas, we can assure the existence of a relativistic temperature, which transforms as: $T = \gamma^{-1} T_0$, where T, T_0 are the temperatures at the moving system and at the rest frame respectively and $\gamma = 1/\sqrt{1 - u^2/c^2}$, where u represents the velocity between both systems. With this result, we are able to analyze the black-body distribution of $2.7K$ radiation background.

$$n(w, T_o, u)\,\mathrm{d}w = \frac{wkT_o\sqrt{1-\frac{u^2}{c^2}}}{2\pi^2c^2u\hbar} \ln \frac{1-\exp-\left(\frac{\hbar w\sqrt{1+\frac{u}{c}}}{kT_o\sqrt{1-\frac{u}{c}}}\right)}{1-\exp-\left(\frac{\hbar w\sqrt{1-\frac{u}{c}}}{kT_o\sqrt{1+\frac{u}{c}}}\right)}\,\mathrm{d}w$$

which is proportional to the excitation rate of the Unruh–DeWitt detector [6, 7]. Landsberg and Matsas [6] assert that since the result is not of a Planckian form, it is impossible to define a temperature transformation law.

$$N = V\int_0^\infty n(w, T_o, u)\,\mathrm{d}w = \frac{2\zeta(3)}{\pi^2}\left(\frac{T_o}{\hbar c}\right)^3,$$

$$E = V\int_0^\infty \hbar w n(w, T_o, u)\,\mathrm{d}w = \frac{4}{c}V\sigma T_o^4\gamma^2\left(1+\frac{u^2}{3c^2}\right).$$

$$E = \gamma E_o\left(1+\frac{u^2}{3c^2}\right) \qquad E = \gamma E_o\left(1+\frac{u^2}{3c^2}\right)$$

the PE transformation law of the energy is consistent with the excitation rate of the Unruh–DeWitt detector. Thus, it will always be possible to define a temperature transformation law which coincides with the PE proposal.

Figure 3: P-E transformation and the Unruh-DeWitt detector

The corrected theory will be called the Redefined Relativistic Thermodynamics [RRT] [23] and it needs to take into account the Nakamura concept of the volume [18] which permits us to define the energy momentum of a system in a covariant form.

If a particular choice of the volume is made, one of the different Rohrlich proposals [9] of the relativistic transformation law of the heat appears as a consistent theory.

Indeed, three different consistent models can be deduced from this theory and they are described by considering the instantaneity in a system and the definition of the rest frame of the volume.

By using two blackbodies with different velocities, it can be shown that the van Kampen theory [19] do not conserve the heat and for this reason is must be discarded.

By using the Henry temperature [20], [21] and the excitation rate of the Unruh DeWitt detector [20], [21], [22] and using the RRT, a relativistic thermometer can be proposed.

The corrected theory will be called the Redefined Relativistic Thermodynamics [RRT] [23] and it needs to take into account the Nakamura concept of the volume [18] which permits us to define the energy momentum of a system in a covariant form.

If a particular choice of the volume is made, one of the different Rohrlich proposals [9] of the relativistic transformation law of the heat appears as a consistent theory.

Indeed, three different consistent models can be deduced from this theory and they are described by considering the instantaneity in a system and the definition of the rest frame of the volume.

By using two blackbodies with different velocities, it can be shown that the van Kampen theory [19] do not conserve the heat and for this reason is must be discarded.

By using the Henry temperature [20], [21] and the excitation rate of the Unruh DeWitt detector [20], [21], [22] and using the RRT, a relativistic thermometer can be proposed.

The corrected theory will be called the Redefined Relativistic Thermodynamics [RRT] [23] and it needs to take into account the Nakamura concept of the volume [18] which permits us to define the energy momentum of a system in a covariant form.

If a particular choice of the volume is made, one of the different Rohrlich proposals [9] of the relativistic transformation law of the heat appears as a consistent theory.

Indeed, three different consistent models can be deduced from this theory and they are described by considering the instantaneity in a system and the definition of the rest frame of the volume.

By using two blackbodies with different velocities, it can be shown that the van Kampen theory [19] do not conserve the heat and for this reason is must be discarded.

By using the Henry temperature [20], [21] and the excitation rate of the Unruh DeWitt detector [20], [21], [22] and using the RRT, a relativistic thermometer can be proposed.

The corrected theory will be called the Redefined Relativistic Thermodynamics [RRT] [23] and it needs to take into account the Nakamura concept of the volume [18] which permits us to define the energy momentum of a system in a covariant form.

If a particular choice of the volume is made, one of the different Rohrlich proposals [9] of the relativistic transformation law of the heat appears as a consistent theory.

Indeed, three different consistent models can be deduced from this theory and they are described by considering the instantaneity in a system and the definition of the rest frame of the volume.

By using two blackbodies with different velocities, it can be shown that the van Kampen theory [19] do not conserve the heat and for this reason is must be discarded.

By using the Henry temperature [20], [21] and the excitation rate of the Unruh DeWitt detector [20], [21], [22] and using the RRT, a relativistic thermometer can be proposed.

The corrected theory will be called the Redefined Relativistic Thermodynamics [RRT] [23] and it needs to take into account the Nakamura concept of the volume [18] which permits us to define the energy momentum of a system in a covariant form.

If a particular choice of the volume is made, one of the different Rohrlich proposals [9] of the relativistic transformation law of the heat appears as a consistent theory.

Indeed, three different consistent models can be deduced from this theory and they are described by considering the instantaneity in a system and the definition of the rest frame of the volume.

By using two blackbodies with different velocities, it can be shown that the van Kampen theory [19] do not conserve the heat and for this reason is must be discarded.

By using the Henry temperature [20], [21] and the excitation rate of the Unruh DeWitt detector [20], [21], [22] and using the RRT, a relativistic thermometer can be proposed.

Nevertheless, by using a different method, Lineweaver [24] has shown that the direction of the reference frame such that the black-body which carries on the $2.7K$ radiation background is at rest, corresponds to the Hydra-Centaur direction with a speed of $627km/sec$. Therefore, since the speed of the reference frame is $627km/sec$, which is very small in comparison with the speed of the light, we can assure that the temperature $2.7K$ is correct; that is: it represents the temperature of the cosmic microwave background radiation (CMB or CMBR) at rest since $\gamma \simeq 1$. If the detector is put in the Hydra-Centaur direction with a speed of $627km/sec$, the particle number density will have to be independent of the angle, that is, the dipolar effect should disappear.

Einstein's centennial commemorative congress at the University of Illinois at Carbondale, Dirac [25] claimed that due to the dipole anisotropy of the radiation background, it will be possible to find the velocity of such a frame.

Nevertheless, by using a different method, Lineweaver [24] has shown that the direction of the reference frame such that the black-body which carries on the $2.7K$ radiation background is at rest, corresponds to the Hydra-Centaur direction with a speed of $627km/sec$. Therefore, since the speed of the reference frame is $627km/sec$, which is very small in comparison with the speed of the light, we can assure that the temperature $2.7K$ is correct; that is: it represents the temperature of the cosmic microwave background radiation (CMB or CMBR) at rest since $\gamma \simeq 1$. If the detector is put in the Hydra-Centaur direction with a speed of $627km/sec$, the particle number density will have to be independent of the angle, that is, the dipolar effect should disappear.

Einstein's centennial commemorative congress at the University of Illinois at Carbondale, Dirac [25] claimed that due to the dipole anisotropy of the radiation background, it will be possible to find the velocity of such a frame.

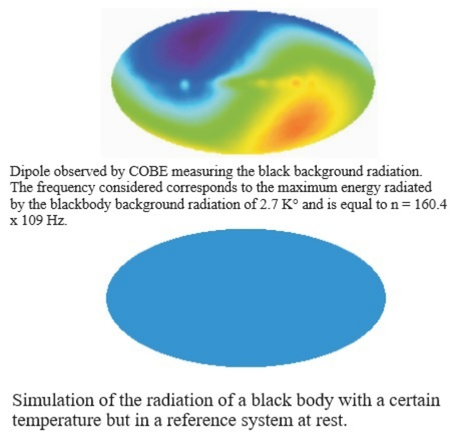

Figure 4: Balescu's classification of the different proposal about relativistic transformation in Thermodynamics

By extrapolation, we will be able to determine the reference frame where the Big-Bang occurred and it will represent a privileged frame contradicting the general theory of relativity.

In the same congress, Differing from Dirac's argument, Wigner [25] commented that general relativity just assures the equivalence of the laws of the nature in each reference frame and it indicates nothing about the initial conditions and so the discovery of a reference frame at rest with the Big-Bang will not contradict the general theory of relativity.

The second comment corresponds to note that Henry et al [21] by the time when the $2.7K$ radiation background was discovered, assured that the sole effect of a uniform motion through black-body cavity is to introduce an effective temperature which replaces the rest frame cavity temperature for each angle. If not, Wigner will be wrong and Dirac will possess the truth [25].

By extrapolation, we will be able to determine the reference frame where the Big-Bang occurred and it will represent a privileged frame contradicting the general theory of relativity.
In the same congress, Differing from Dirac's argument, Wigner [25] commented that general relativity just assures the equivalence of the laws of the nature in each reference frame and it indicates nothing about the initial conditions and so the discovery of a reference frame at rest with the Big-Bang will not contradict the general theory of relativity.
The second comment corresponds to note that Henry et al [21] by the time when the $2.7K$ radiation background was discovered, assured that the sole effect of a uniform motion through black-body cavity is to introduce an effective temperature which replaces the rest frame cavity temperature for each angle. If not, Wigner will be wrong and Dirac will possess the truth [25].

By extrapolation, we will be able to determine the reference frame where the Big-Bang occurred and it will represent a privileged frame contradicting the general theory of relativity.
In the same congress, Differing from Dirac's argument, Wigner [25] commented that general relativity just assures the equivalence of the laws of the nature in each reference frame and it indicates nothing about the initial conditions and so the discovery of a reference frame at rest with the Big-Bang will not contradict the general theory of relativity.
The second comment corresponds to note that Henry et al [21] by the time when the $2.7K$ radiation background was discovered, assured that the sole effect of a uniform motion through black-body cavity is to introduce an effective temperature which replaces the rest frame cavity temperature for each angle. If not, Wigner will be wrong and Dirac will possess the truth [25].

The article is organized as follows:

in section 2, the Redefined Relativistic Thermodynamics is presented giving the new definitions of the different quantities and obtaining the three principal proposals; namely: The PE proposal, the O proposal and the L proposal.

In section 3, the Statistical Mechanics is described by its corresponding relativistic transformation laws.

In section 4, a brief description of the Einstein´s Dual Theory is exposed including the role that plays by the proper time.

In section 5, the RRT is related to Einstein's dual theory, concluding that PE proposal may be the most compatible with the Einstein´s Dual Theory.

Some concluding remarks are done in section 6.

The article is organized as follows:
in section 2, the Redefined Relativistic Thermodynamics is presented giving the new definitions of the different quantities and obtaining the three principal proposals; namely: The PE proposal, the O proposal and the L proposal.
In section 3, the Statistical Mechanics is described by its corresponding relativistic transformation laws.
In section 4, a brief description of the Einstein´s Dual Theory is exposed including the role that plays by the proper time.
In section 5, the RRT is related to Einstein's dual theory, concluding that PE proposal may be the most compatible with the Einstein´s Dual Theory.
Some concluding remarks are done in section 6.

The article is organized as follows:

in section 2, the Redefined Relativistic Thermodynamics is presented giving the new definitions of the different quantities and obtaining the three principal proposals; namely: The PE proposal, the O proposal and the L proposal.

In section 3, the Statistical Mechanics is described by its corresponding relativistic transformation laws.

In section 4, a brief description of the Einstein´s Dual Theory is exposed including the role that plays by the proper time.

In section 5, the RRT is related to Einstein's dual theory, concluding that PE proposal may be the most compatible with the Einstein´s Dual Theory.

Some concluding remarks are done in section 6.

The article is organized as follows:
in section 2, the Redefined Relativistic Thermodynamics is presented giving the new definitions of the different quantities and obtaining the three principal proposals; namely: The PE proposal, the O proposal and the L proposal.
In section 3, the Statistical Mechanics is described by its corresponding relativistic transformation laws.
In section 4, a brief description of the Einstein´s Dual Theory is exposed including the role that plays by the proper time.
In section 5, the RRT is related to Einstein's dual theory, concluding that PE proposal may be the most compatible with the Einstein´s Dual Theory.
Some concluding remarks are done in section 6.

The article is organized as follows:

in section 2, the Redefined Relativistic Thermodynamics is presented giving the new definitions of the different quantities and obtaining the three principal proposals; namely: The PE proposal, the O proposal and the L proposal.

In section 3, the Statistical Mechanics is described by its corresponding relativistic transformation laws.

In section 4, a brief description of the Einstein´s Dual Theory is exposed including the role that plays by the proper time.

In section 5, the RRT is related to Einstein's dual theory, concluding that PE proposal may be the most compatible with the Einstein´s Dual Theory.

Some concluding remarks are done in section 6.

The article is organized as follows:
in section 2, the Redefined Relativistic Thermodynamics is presented giving the new definitions of the different quantities and obtaining the three principal proposals; namely: The PE proposal, the O proposal and the L proposal.
In section 3, the Statistical Mechanics is described by its corresponding relativistic transformation laws.
In section 4, a brief description of the Einstein´s Dual Theory is exposed including the role that plays by the proper time.
In section 5, the RRT is related to Einstein's dual theory, concluding that PE proposal may be the most compatible with the Einstein´s Dual Theory.
Some concluding remarks are done in section 6.

2- Relativistic Transformation Laws of Thermodynamic Quantities

From the beginning of Special Relativity, Fermi noticed that the sum of vectorial quantities along a $4-$volume or $3-$hypersurface is not well-defined quantity unless some divergences vanish [37], [38], [39], [9], [40], [41]. This result obligates to well define the quantities that we will integrate, and the $4-$volume or the $3-$hypersurface, etc.

Therefore, first of all, let's give a brief overview of Redefined Relativistic Thermodynamics [RRT]. Mathematically, as Nakamura [18] proposed, we can describe the volume as follows: a three dimensional flat plane is defined as a set of events that satisfies

$$\omega^{\mu} x_{\mu} = 0, \tag{II-1}$$

where ω^{μ} represents the time-like unit vector that defines the direction of the three-dimensional instantaneous space in the four-dimensional space.

2- Relativistic Transformation Laws of Thermodynamic Quantities

From the beginning of Special Relativity, Fermi noticed that the sum of vectorial quantities along a $4-$volume or $3-$hypersurface is not well-defined quantity unless some divergences vanish [37], [38], [39], [9], [40], [41]. This result obligates to well define the quantities that we will integrate, and the $4-$volume or the $3-$hypersurface, etc.
Therefore, first of all, let's give a brief overview of Redefined Relativistic Thermodynamics [RRT]. Mathematically, as Nakamura [18] proposed, we can describe the volume as follows: a three dimensional flat plane is defined as a set of events that satisfies

$$\omega^{\mu} x_{\mu} = 0, \tag{II-1}$$

where ω^{μ} represents the time-like unit vector that defines the direction of the three-dimensional instantaneous space in the four-dimensional space.

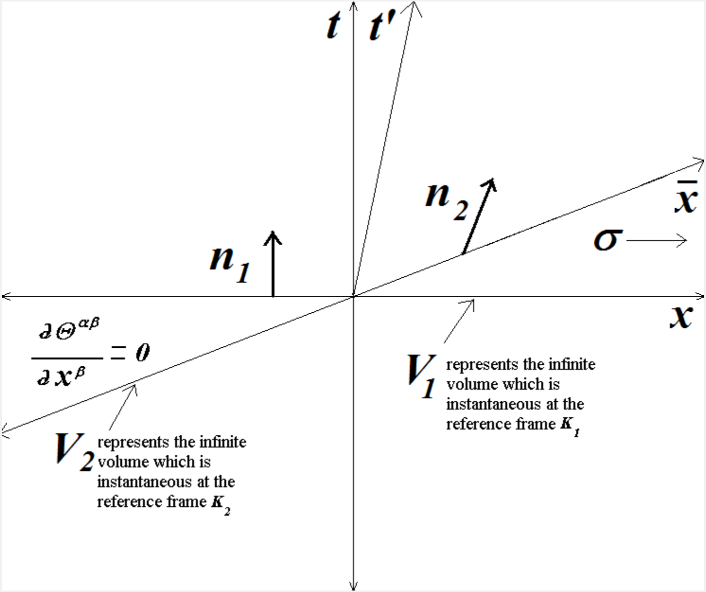

Figure 5: Interacting Particles

The volume $V_K(w)$ is defined as the intersection of this flat plane and the world tube of the object. If we consider a frame K and the volume V_{rest} in its rest frame K_{rest}, the unit vector $\mathbf{u}$ represents the motion between K and K_{rest}. The volume now is defined as

$$V(w) = \frac{V_{rest}}{u^\mu \omega_\mu}, \quad \text{(II-2)}$$

and the corresponding $4-$vector volume

$$V^\mu = \frac{\omega^\mu V_{rest}}{u^\nu \omega_\nu}. \quad \text{(II-3)}$$

The volume $V_K(w)$ is defined as the intersection of this flat plane and the world tube of the object. If we consider a frame K and the volume V_{rest} in its rest frame K_{rest}, the unit vector $\mathbf{u}$ represents the motion between K and K_{rest}. The volume now is defined as

$$V(w) = \frac{V_{rest}}{u^{\mu}\omega_{\mu}}, \tag{II-2}$$

and the corresponding $4-$vector volume

$$V^{\mu} = \frac{\omega^{\mu} V_{rest}}{u^{\nu}\omega_{\nu}}. \tag{II-3}$$

2.1- General Thermodynamics

By generalizing to the other components, we put

$$\xi^{\mu} = \frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}} E_{rest}, \tag{II-4}$$

and

$$d\Theta^{\mu} = \frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}} dQ_{rest}, \tag{II-5}$$

to arrive at

$$d\xi^{\mu} = d\Theta^{\mu} - \mathbb{P} dV^{\mu} = d\Theta^{\mu} - dW^{\mu}, \tag{II-6}$$

with

$$d\xi^{\mu} = \omega^{\mu} \frac{dE_{rest}}{u^{\lambda}\omega_{\lambda}}, \quad d\Theta^{\mu} = \omega^{\mu} \frac{dQ_{rest}}{u^{\lambda}\omega_{\lambda}} \quad \text{and} \quad dW^{\mu} = \omega^{\mu} \frac{\mathbb{P} dV_{rest}}{u^{\lambda}\omega_{\lambda}}. \tag{II-7}$$

2.1- General Thermodynamics

By generalizing to the other components, we put

$$\xi^\mu = \frac{\omega^\mu}{u^\lambda \omega_\lambda} E_{rest}, \tag{II-4}$$

and

$$d\Theta^\mu = \frac{\omega^\mu}{u^\lambda \omega_\lambda} dQ_{rest}, \tag{II-5}$$

to arrive at

$$d\xi^\mu = d\Theta^\mu - \mathbb{P} dV^\mu = d\Theta^\mu - dW^\mu, \tag{II-6}$$

with

$$d\xi^\mu = \omega^\mu \frac{dE_{rest}}{u^\lambda \omega_\lambda}, \quad d\Theta^\mu = \omega^\mu \frac{dQ_{rest}}{u^\lambda \omega_\lambda} \quad \text{and} \quad dW^\mu = \omega^\mu \frac{\mathbb{P} dV_{rest}}{u^\lambda \omega_\lambda}. \tag{II-7}$$

If we define the $4-$vector temperature as

$$T^{\mu} = \frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}} T_{rest}, \tag{II-8}$$

The question now is how to express a covariant relativistic thermodynamics independent of the choice of the volume; the answer consists of noticing that if we define the $4-$vector G^{μ} as

$$G^{\mu} = \left[(\mathbb{P} + e_{rest})u^{\mu}u^{\nu} - (\mathbb{P} + e_{rest})g^{\mu\nu}\right]\frac{V_{rest}\omega_{\nu}}{u^{\lambda}\omega_{\lambda}}$$

$$G^{\mu} = (\mathbb{P}V_{rest} + E_{rest})u^{\mu} - (\mathbb{P}V_{rest} + E_{rest})\frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}}. \tag{II-9}$$

If we define the $4-$vector temperature as

$$T^{\mu} = \frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}} T_{rest}, \qquad \text{(II-8)}$$

The question now is how to express a covariant relativistic thermodynamics independent of the choice of the volume; the answer consists of noticing that if we define the $4-$vector G^{μ} as

$$G^{\mu} = \left[(\mathbb{P} + e_{rest})u^{\mu}u^{\nu} - (\mathbb{P} + e_{rest})g^{\mu\nu}\right] \frac{V_{rest}\omega_{\nu}}{u^{\lambda}\omega_{\lambda}}$$

$$G^{\mu} = (\mathbb{P}V_{rest} + E_{rest})u^{\mu} - (\mathbb{P}V_{rest} + E_{rest})\frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}}. \qquad \text{(II-9)}$$

And then we construct the $4-$vector:

$$P^{\mu} - G^{\mu} = (\mathbb{P}V_{rest} + E_{rest})u^{\mu} - \mathbb{P}V_{rest}\frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}} - (\mathbb{P}V_{rest} + E_{rest})u^{\mu}$$

$-(\mathrm{PV}_{rest} + E_{rest})\frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}}$

$$P^{\mu} - G^{\mu} = \frac{\omega^{\mu}}{u^{\lambda}\omega_{\lambda}}E_{rest} = \xi^{\mu}. \qquad \text{(II-10)}$$

Therefore, we can assure that for any chosen volume, a covariant relativistic thermodynamics that we will call the Redefined Relativistic Thermodynamics [RRT], can be described by the following relations.

And then we construct the $4-$vector:

$$P^\mu - G^\mu = (\mathbb{P}V_{rest} + E_{rest})u^\mu - \mathbb{P}V_{rest}\frac{\omega^\mu}{u^\lambda\omega_\lambda} - (\mathbb{P}V_{rest} + E_{rest})u^\mu$$

$$-(\mathsf{PV}_{rest} + E_{rest})\tfrac{\omega^\mu}{u^\lambda\omega_\lambda}$$

$$P^\mu - G^\mu = \frac{\omega^\mu}{u^\lambda\omega_\lambda}E_{rest} = \xi^\mu. \tag{II-10}$$

Therefore, we can assure that for any chosen volume, a covariant relativistic thermodynamics that we will call the Redefined Relativistic Thermodynamics [RRT], can be described by the following relations.

$$d\xi^{\mu} = d\Theta^{\mu} - \mathbb{P}dW^{\mu}, \tag{II-11}$$

where

$$d\xi^{\mu} = \omega^{\mu}\frac{dE_{rest}}{u^{\lambda}\omega_{\lambda}}, \quad d\Theta^{\mu} = \omega^{\mu}\frac{dQ_{rest}}{u^{\lambda}\omega_{\lambda}} \quad \text{and} \quad dW^{\mu} = \omega^{\mu}\frac{\mathbb{P}dV_{rest}}{u^{\lambda}\omega_{\lambda}}, \tag{II-12}$$

$$d\Theta^{\mu} = \omega^{\mu}\frac{dQ_{rest}}{u^{\lambda}\omega_{\lambda}}, \quad T^{\mu} = \omega^{\mu}\frac{T_{rest}}{u^{\lambda}\omega_{\lambda}} \quad \text{and} \quad \beta_{\mu} = \frac{u_{\mu}}{kT_{rest}}, \tag{II-13}$$

and with the entropy expressed as

$$dS = \beta_{\mu}\frac{V_{rest}\omega_{\nu}}{u^{\lambda}\omega_{\lambda}}dT^{\mu\nu} + \beta_{\mu}\mathbb{P}dV^{\mu}. \tag{II-14}$$

2.2- Case 1 or Rohrlich-Ott-Gamba-like proposal [ROG]:

If we want to deal with the volume which represents all the events that simultaneously occur in the rest frame of the volume and if we look this volume from the frame K, we have

$$\omega^\mu = \left(\gamma, \gamma\frac{\mathbf{u}}{c}\right) \qquad and \qquad u^\mu = \left(\gamma, \gamma\frac{\mathbf{u}}{c}\right). \tag{II-15}$$

Therefore,

$$u^\mu \omega_\mu = 1. \tag{II-16}$$

The considered volume is

$$V_{ROG} = \frac{V_{rest}}{u^\mu \omega_\mu} = V_{rest}, \tag{II-17}$$

and the corresponding $4-$vector volume is represented by

$$V^\mu_{ROG} = \frac{\omega^\mu V_{rest}}{u^\nu \omega_\nu} = \omega^\mu V_{rest} = u^\mu V_{rest} = \left(\gamma, \gamma\frac{\mathbf{u}}{c}\right) V_{rest}. \tag{II-18}$$

ROG

Therefore, in this case, the differential of the volume is:

$$dV^{\mu}_{ROG} = \frac{w^{\mu} dV_{rest}}{u^{\nu}\omega_{\nu}} = \omega^{\mu} dV_{rest} = u^{\mu} dV_{rest} = \left(\gamma, \gamma\frac{\mathbf{u}}{c}\right) dV_{rest}. \tag{II-19}$$

which represents the 4-vector differential of the volume that Gamba [41] has used in order to clarify the electromagnetic controversy of the $4/3$ term. If we notice that in the ROG case, G^{μ} vanishes, that is,

$$G^{\mu} = (\mathbb{P}V_{rest} + E_{rest})u^{\mu} - (\mathbb{P}V_{rest} + E_{rest})u^{\mu} = 0, \tag{II-20}$$

we have $P^{\mu}_{RG} = \xi^{\mu}_{RG}$.

2.2.1- Thermodynamics of the ROG-like proposal

The thermodynamical relations are obtained just by defining the volume. The Rohrlich-Ott-Gamba proposal is obtained by choosing the ROG volume. A corrected redefined relativistic thermodynamics is also found by using the PE volume. The interesting fact is that for any other choice of the volume we will obtain the thermodynamical relations. It can be thought that this new theory does not contribute to thermodynamics since we have relations of the form $BdE_{rest} = BdQ_{rest} - B\mathbb{P}dV_{rest}$ that can be always simplified to $dE_{rest} = dQ_{rest} - \mathbb{P}dV_{rest}$, that is, the thermodynamics in the rest frame.

Nevertheless, when two systems are considered to interact this formalism will be useful. This will be applied to the study of two blackbodies interacting with the same initial conditions but with different motions.

2.2.1- Thermodynamics of the ROG-like proposal

The thermodynamical relations are obtained just by defining the volume. The Rohrlich-Ott-Gamba proposal is obtained by choosing the ROG volume. A corrected redefined relativistic thermodynamics is also found by using the PE volume. The interesting fact is that for any other choice of the volume we will obtain the thermodynamical relations. It can be thought that this new theory does not contribute to thermodynamics since we have relations of the form $BdE_{rest} = BdQ_{rest} - B\mathbb{P}dV_{rest}$ that can be always simplified to $dE_{rest} = dQ_{rest} - \mathbb{P}dV_{rest}$, that is, the thermodynamics in the rest frame.
Nevertheless, when two systems are considered to interact this formalism will be useful. This will be applied to the study of two blackbodies interacting with the same initial conditions but with different motions.

Before finishing this section, it has to be pointed out that the validity of the theory must satisfy some constraints. The idea can be summarized by the following sentences due to Nakamura [18]:"This means the change is slow enough to be able to regard objects in interest is in equilibrium all the time. There can be another requirement in relativistic thermodynamics; the change must be much slower than the transit time scale of the light across the object. However, this requirement is satisfied when we can regard the process as adiabatic, since an object cannot be in equilibrium within a time scale shorter than the transit time of light." A summary of the ROG is represented in Table 1.

V^0	$\mathbb{P}$	S	T^0	$đ\Theta^0$	ξ^0	F^0
γV_o	$\mathbb{P}_o$	S_o	γT_o	$\gamma đQ_o$	γE_o	γF_o

Table 1

Before finishing this section, it has to be pointed out that the validity of the theory must satisfy some constraints. The idea can be summarized by the following sentences due to Nakamura [18]:"This means the change is slow enough to be able to regard objects in interest is in equilibrium all the time. There can be another requirement in relativistic thermodynamics; the change must be much slower than the transit time scale of the light across the object. However, this requirement is satisfied when we can regard the process as adiabatic, since an object cannot be in equilibrium within a time scale shorter than the transit time of light." A summary of the ROG is represented in Table 1.

V^0	$\mathbb{P}$	S	T^0	$đ\Theta^0$	ξ^0	F^0
γV_o	$\mathbb{P}_o$	S_o	γT_o	$\gamma đQ_o$	γE_o	γF_o

Table 1

Before finishing this section, it has to be pointed out that the validity of the theory must satisfy some constraints. The idea can be summarized by the following sentences due to Nakamura [18]:"This means the change is slow enough to be able to regard objects in interest is in equilibrium all the time. There can be another requirement in relativistic thermodynamics; the change must be much slower than the transit time scale of the light across the object. However, this requirement is satisfied when we can regard the process as adiabatic, since an object cannot be in equilibrium within a time scale shorter than the transit time of light." A summary of the ROG is represented in Table 1.

V^0	$\mathbb{P}$	S	T^0	$đ\Theta^0$	ξ^0	F^0
γV_o	$\mathbb{P}_o$	S_o	γT_o	$\gamma đ Q_o$	γE_o	γF_o

Table 1

2.3- Case 2 or Planck-Einstein-like proposal [PE]

Let us now consider a volume generated by a Lorentz contraction in K of a body whose volume V_{rest} is at rest in K_{rest} . This volume is equal to

$$V_K = \gamma^{-1} V_{rest}. \quad \text{(II-21)}$$

in K. As we noticed before, all the points of this volume are simultaneous in K and, consequently, we can consider it as a volume at rest in K.

In the Nakamura formalism, the different $4-$vectors will be described in K by

$$\omega^\mu = (1, \mathbf{0}) \qquad and \qquad u^\mu = \left(\gamma, \gamma \frac{\mathbf{u}}{c}\right). \quad \text{(II-22)}$$

2.3- Case 2 or Planck-Einstein-like proposal [PE]

Let us now consider a volume generated by a Lorentz contraction in K of a body whose volume V_{rest} is at rest in K_{rest} . This volume is equal to

$$V_K = \gamma^{-1} V_{rest}. \tag{II-21}$$

in K. As we noticed before, all the points of this volume are simultaneous in K and, consequently, we can consider it as a volume at rest in K.

In the Nakamura formalism, the different $4-$vectors will be described in K by

$$\omega^{\mu} = (1, \mathbf{0}) \qquad and \qquad u^{\mu} = \left(\gamma, \gamma\frac{\mathbf{u}}{c}\right). \tag{II-22}$$

Therefore,

$$u^{\mu}\omega_{\mu} = \gamma. \tag{II-23}$$

The considered volume is

$$V_{K_{rest}} = \frac{V_{rest}}{u^{\mu}\omega_{\mu}} = \frac{V_{rest}}{\gamma} = \gamma^{-1}V_{rest}, \tag{II-24}$$

and the corresponding $4-$vector volume is represented by

$$V^{\mu}_{K_{rest}} = \frac{\omega^{\mu}V_{rest}}{u^{\nu}\omega_{\nu}} = \omega^{\mu}\frac{V_{rest}}{\gamma} = (1,0)\,\frac{V_{rest}}{\gamma}. \tag{II-25}$$

Therefore, in this case, the differential of the volume is:

$$dV^{\mu}_{K_{rest}} = \frac{\omega^{\mu}dV_{rest}}{u^{\nu}\omega_{\nu}} = \omega^{\mu}\frac{dV_{rest}}{\gamma} = (1,0)\,\frac{dV_{rest}}{\gamma}. \tag{II-26}$$

Therefore,

$$u^\mu \omega_\mu = \gamma. \quad \text{(II-23)}$$

The considered volume is

$$V_{K_{rest}} = \frac{V_{rest}}{u^\mu \omega_\mu} = \frac{V_{rest}}{\gamma} = \gamma^{-1} V_{rest}, \quad \text{(II-24)}$$

and the corresponding $4-$vector volume is represented by

$$V^\mu_{K_{rest}} = \frac{\omega^\mu V_{rest}}{u^\nu \omega_\nu} = \omega^\mu \frac{V_{rest}}{\gamma} = (1,0)\,\frac{V_{rest}}{\gamma}. \quad \text{(II-25)}$$

Therefore, in this case, the differential of the volume is:

$$dV^\mu_{K_{rest}} = \frac{\omega^\mu dV_{rest}}{u^\nu \omega_\nu} = \omega^\mu \frac{dV_{rest}}{\gamma} = (1,0)\,\frac{dV_{rest}}{\gamma}. \quad \text{(II-26)}$$

Therefore,

$$u^{\mu}\omega_{\mu} = \gamma. \tag{II-23}$$

The considered volume is

$$V_{K_{rest}} = \frac{V_{rest}}{u^{\mu}\omega_{\mu}} = \frac{V_{rest}}{\gamma} = \gamma^{-1}V_{rest}, \tag{II-24}$$

and the corresponding $4-$vector volume is represented by

$$V^{\mu}_{K_{rest}} = \frac{\omega^{\mu}V_{rest}}{u^{\nu}\omega_{\nu}} = \omega^{\mu}\frac{V_{rest}}{\gamma} = (1,0)\,\frac{V_{rest}}{\gamma}. \tag{II-25}$$

Therefore, in this case, the differential of the volume is:

$$dV^{\mu}_{K_{rest}} = \frac{\omega^{\mu}dV_{rest}}{u^{\nu}\omega_{\nu}} = \omega^{\mu}\frac{dV_{rest}}{\gamma} = (1,0)\,\frac{dV_{rest}}{\gamma}. \tag{II-26}$$

PE

Therefore,

$$u^{\mu}\omega_{\mu} = \gamma. \tag{II-23}$$

The considered volume is

$$V_{K_{rest}} = \frac{V_{rest}}{u^{\mu}\omega_{\mu}} = \frac{V_{rest}}{\gamma} = \gamma^{-1}V_{rest}, \tag{II-24}$$

and the corresponding $4-$vector volume is represented by

$$V^{\mu}_{K_{rest}} = \frac{\omega^{\mu}V_{rest}}{u^{\nu}\omega_{\nu}} = \omega^{\mu}\frac{V_{rest}}{\gamma} = (1,0)\,\frac{V_{rest}}{\gamma}. \tag{II-25}$$

Therefore, in this case, the differential of the volume is:

$$dV^{\mu}_{K_{rest}} = \frac{\omega^{\mu}dV_{rest}}{u^{\nu}\omega_{\nu}} = \omega^{\mu}\frac{dV_{rest}}{\gamma} = (1,0)\,\frac{dV_{rest}}{\gamma}. \tag{II-26}$$

2.3.1- Thermodynamics of the Planck-Einstein-like proposal

Let us present a special resume of the theory when the PE volume is chosen in a not covariant form. Indeed, the last formalism keeps the invariance of the form in thermodynamics by defining the following quantities: the volume $V = V^0_{PE} = \gamma^{-1} V_{rest}$, the redefined energy ξ

$$\xi = E - \gamma(E_{rest} + \mathbb{P}V_{rest})\frac{u^2}{c^2} = \xi^0 = \gamma^{-1} E_{rest}, \qquad \text{(II-27)}$$

where E represents the relativistic PE transformed internal energy. Table 2 describes the more important transformed quantities: (F represents the Helmholtz free energy)

V	$\mathbb{P}$	S	T	$d\Theta$	ξ	F
$\frac{V_{rest}}{\gamma}$	$\mathbb{P}_{rest}$	S_{rest}	$\frac{T_{rest}}{\gamma}$	$\frac{dQ_{rest}}{\gamma}$	$\frac{\xi_{rest}}{\gamma} = \frac{E_{rest}}{\gamma}$	$\frac{F_{rest}}{\gamma}$

Table 2

PE

The difference with the "renormalized" relativistic thermodynamics consists of noticing that the heat has been redefined as the redefined heat and the work has kept its form without adding to it the bulk energy. Indeed, the work transforms as $dW = \mathbb{P}\gamma^{-1}dV_{rest}$, where $\mathbb{P}$ represents the pressure. With this definition, we obtain all the thermodynamical relations, as for example, the first law, that is,

$$d\xi^0 = d\Theta^0 - \mathbb{P}dV^0. \quad \text{(II-28)}$$

or simply

$$d\xi = d\Theta - \mathbb{P}dV. \quad \text{(II-28)}$$

Landsberg-like Proposal [L]

If we want to measure at K and the rest volume is in K

$$\omega^\mu = (1, \mathbf{0}) \qquad and \qquad u^\mu = (1, \mathbf{0}) \tag{II-29}$$

$$u^\mu \omega_\mu = 1, \; V = \frac{V_{rest}}{u^\mu \omega_\mu} = V_{rest}, \; V^\mu = \frac{w^\mu}{u^\mu \omega_\mu} V_{rest} = (1, 0)\, V_{rest}, \tag{II-30}$$

$$dV^\mu = \frac{\omega^\mu}{u^\mu \omega_\mu} dV_{rest} = (1, 0)\, dV_{rest}. \tag{II-31}$$

Table 3 describes the more important transformed quantities: (F represents the Helmholtz free energy).

V	$\mathbb{P}$	S	T	$d\Theta$	ξ	F
V_{rest}	$\mathbb{P}_{rest}$	S_{rest}	T_{rest}	dQ_{rest}	$\xi_{rest} = E_{rest}$	F_{rest}

Table 3

Landsberg-like Proposal [L]

If we want to measure at K and the rest volume is in K

$$\omega^{\mu} = (1, \mathbf{0}) \qquad and \qquad u^{\mu} = (1, \mathbf{0}) \tag{II-29}$$

$$u^{\mu}\omega_{\mu} = 1, \; V = \frac{V_{rest}}{u^{\mu}\omega_{\mu}} = V_{rest}, \; V^{\mu} = \frac{w^{\mu}}{u^{\mu}\omega_{\mu}} V_{rest} = (1, 0)\, V_{rest}, \tag{II-30}$$

$$dV^{\mu} = \frac{\omega^{\mu}}{u^{\mu}\omega_{\mu}} dV_{rest} = (1, 0)\, dV_{rest}. \tag{II-31}$$

Table 3 describes the more important transformed quantities: (F represents the Helmholtz free energy).

V	$\mathbb{P}$	S	T	$d\Theta$	ξ	F
V_{rest}	$\mathbb{P}_{rest}$	S_{rest}	T_{rest}	dQ_{rest}	$\xi_{rest} = E_{rest}$	F_{rest}

Table 3

L

Of course this L proposal just consider that we are measuring all the thermodynamics quantities in the frame where the thermodynamic system (the volume) is at rest. Many authors consider that this is the way we have to deal with thermodynamic system, however when we are dealing with a system composed of a Mixing of gases with different relative velocities [42], it is necessary to use the ROG or the PE proposals, or to deal with the center of mass of the whole system as we wiil see later.

Statistical Mechanics and the Relativistic Transformation Laws

On the other hand, It is necessary to deal with Relativistic Statistical Mechanics and Canonical transformation. Curie et al [26] and later on Balescu and Kotera [27] have demonstrated that the distribution function in a moving system, with velocity u in the $x-$axis with respect the rest frame of the system, can be expressed as [28], [29]:

$$f(q,p;s) = e^{[K_1]s} f\left(q,p,0\right) \tag{III-1}$$

where q and p represent the canonical variables, $u = tanhs$ and K_1 is the boost generator in the $x-$axis. Balescu [27], as we noticed above, partially used this result but he directly did not calculate the distribution function. In order to solve the last equation, let us firstly expose some important results.

Statistical Mechanics and the Relativistic Transformation Laws

The exponential of the boost generator applied to a function A, can be expressed as:

$$e^{[K_1]s}A = A + s\,[A, K_1] + \frac{1}{2!}s^2\,[[A, K_1]\,, K_1] + ... \qquad \text{(III-2)}$$

Then, Balescu and Kotera, starting from

$$f(q, p, 0) = e^{\frac{1}{kT_0}[F(To,Vo,0)-H(q,p)]}, \qquad \text{(III-3)}$$

where $[F(To, Vo, 0)$ represents the Helmholtz free energy in the rest frame V_0 at temperature To, arrive at

$$f(q, p, s) = e^{[K_1]s}e^{\frac{1}{kT_0}[F(To,Vo,0)-H(q,p)]}. \qquad \text{(III-4)}$$

Finally,

$$f(q, p, s) = e^{\frac{1}{kT}[F-\Pi(q,p)]}, \qquad \text{(III-5)}$$

Statistical Mechanics and the Relativistic Transformation Laws

with

$$T = \gamma^{-1} T_0, \qquad F = \gamma^{-1} F_0 \qquad and \qquad \Pi = H - P_1 \tanh s. \tag{III-6}$$

That is: the canonical distribution function, which is a probability function of the generalized canonical coordinates q and p, has been derived [30], [31] and it is expressed by:

$$f(q,p,s) = \exp \frac{1}{kT}\left[F - \Pi\right]$$

$$= \exp \frac{\gamma}{kT_{rest}}\left[\gamma^{-1} F_{rest} - \gamma^{-1} H(q,p)\right] = f(q,p,0), \tag{III-7}$$

where $\Pi = H - P_1 c \tanh s$ represents the redefined Hamiltonian and $\tanh s = \frac{u}{c}$. It has to be noticed that this representation of Statistical Mechanics is consistent with the PE proposal.

Statistical Mechanics and the Relativistic Transformation Laws

This is because the probability function is defined by using the instantaneity and the volume as it is proposed in the PE proposal. We can write the partition function as [30]

$$Z(s,T) = \frac{1}{N!h^{3N}} \int d^{3N} dp^{3N} p \exp\left[-\frac{1}{kT}\Pi\right]$$

$$= \frac{1}{N!h^{3N}} \int d^{3N} dp^{3N} p_0 \exp\left[-\frac{\gamma}{kT_0}\frac{H\left(q_0, p_0\right)}{\gamma}\right]$$

$$Z(s,T) = Z(0,T_0) \tag{III-8}$$

which is an invariant.

STMEC

In Quantum Mechanics, in the same way, we arrive at the quantum partition function

$$Q(s) = Tr\exp\left[\frac{1}{kT}\Pi\right] = \sum_i \beta' E_i' \qquad \text{(III-9)}$$

with $\beta' = \gamma/kT$ and $E_i' = E_i - uG_x = \xi_i$, where G_x corresponds to

$$\mathbf{G} = \frac{4}{3}\gamma E_{i0}\frac{\mathbf{u}}{c^2}. \qquad \text{(III-10)}$$

$\mathbf{G}$ corresponds to G^μ for the PE proposal.
The distribution function is [31]

$$f = \frac{1}{Z}e^{-\beta_\mu \Pi^\mu} \qquad \text{(III-11)}$$

with

$$\beta_\mu \Pi^\mu = \frac{1}{kT}\sum_{i=1}^{n}\sqrt{p_i^2c^2 + m^2c^4} - uP. \qquad \text{(III-12)}$$

STMEC

Arrive at Jüttner distribution function

$$f(\mathbf{v}) = \frac{m^3\gamma^5(\mathbf{v})}{\gamma(\mathbf{u})\Upsilon_3\left(\gamma\left(\mathbf{u}\right)T\right)}\exp -\frac{m\gamma(\mathbf{v})}{kT}\left(1-\mathbf{v}\cdot\mathbf{u}\right) \qquad \text{(III-13)}$$

where

$$\Upsilon_3(y) = \frac{4\pi}{h^3}\left(mc\right)^3\frac{K_2\left(\frac{mc^2}{ky}\right)}{mc^2}ky, \qquad \text{(III-14)}$$

with K_2 represents the Bessel function of second rank.

Einstein Dual Theory and the Proper Time

We need to notice that in the Einstein´s Dual Theory [32], [33], [34], [35] and [36] the proper time τ remains invariant, we assume K invariant

$$K = \frac{\pi^2}{2} + mc^2 + \frac{V^2}{2mc^3} + \frac{V\sqrt{c^2\pi^2 + m^2c^4}}{mc^2}, \qquad H_0 = \sqrt{c^2\pi^2 + m^2c^4}, \tag{IV-1}$$

$$\frac{d\mathbf{x}}{d\tau} = \frac{\partial K}{\partial \mathbf{p}} = \frac{H}{mc^2}\left(\frac{c^2\pi}{H_0}\right) = \frac{b}{c}\left(\frac{c^2\mathbf{x}}{H_0}\right) \Rightarrow \frac{d\mathbf{x}}{d\tau} = \frac{b}{c}\frac{d\mathbf{x}}{dt}, \tag{IV-2}$$

$$\pi = \mathbf{p} - \frac{e}{c}\mathbf{A} \tag{IV-3}$$

One particle

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{w}_i, \qquad \frac{d\mathbf{x}_i}{d\tau_i} = \mathbf{u}_i \qquad \frac{d\mathbf{x}_i}{d\tau} = \mathbf{v}_i, \tag{IV-4}$$

where

$$b_i = \sqrt{u_i^2 + c^2} \qquad and \qquad b = \sqrt{U^2 + c^2} \tag{IV-5}$$

$$\frac{\mathbf{w}_i}{c} = \frac{\mathbf{v}_i}{b} = \frac{\mathbf{u}_i}{b_i} \Rightarrow \gamma_i^{-1} = \sqrt{1 - \left(\frac{w_i}{c}\right)^2} = \sqrt{1 - \left(\frac{v_i}{b}\right)^2} = \sqrt{1 - \left(\frac{u_i}{b_i}\right)^2} \tag{IV-6}$$

It has to be noticed that eventually $\mathbf{U}$ may represent the motion of the canonical center of mass as we will see in the next section. The motion of the canonical center of mass is described by

$$\mathbf{X} = \frac{1}{H}\sum_i^N H_i\mathbf{x}_i + \frac{c^2\left(\mathbf{S}\times\mathbf{P}\right)}{H\left(Mc^2 + H\right)}, \tag{IV-7}$$

where $\mathbf{S}$ is the global spin of the system of particles relative to O, the origin. Being

$$\mathbf{V} = \frac{d\mathbf{X}}{dt} \Rightarrow \mathbf{U} = \frac{d\mathbf{X}}{d\tau} = \gamma(\mathbf{V})\mathbf{V} \tag{IV-8}$$

the hamiltonian H of many particle is

$$H = \sum_i H_i \quad with \quad H_i = H_{i0} + V_i = \sqrt{c^2\pi_i^2 + m_i^2 c^4} + V_i \tag{IV-9}$$

where

$$\pi_i = \mathbf{p}_i - \frac{e_i}{c}\mathbf{A}_i \quad with \quad \mathbf{A}_i = \sum_{i \neq j} \mathbf{A}_{ij} \quad and \quad V_i = \sum_{i \neq j} V_{ij}. \tag{IV-10}$$

Then,

$$K = \frac{H^2}{2Mc^2} + Mc^2. \tag{IV-11}$$

Quant

For Quantum Mechanics, we have

$$H_D = c\alpha \cdot \pi + mc^2\beta + V_0, \quad with \quad V = \frac{1}{2mc^2}\left[H_0V_0 + V_0H_0\right] \tag{IV-12}$$

The Dirac Hamiltonian as:

$$K_D = \frac{H_D^2}{2mc^2} + \frac{mc^2}{2} = \frac{\pi^2}{2m} + V - \frac{e\hbar\mathbf{\Sigma}\cdot\mathbf{B}}{2mc} + mc^2 + \frac{V_0^2}{2mc^2}. \tag{IV-13}$$

The Proper Time in Redefined Relativistic Thermodynamics

As we have seen in RRT, using the proper time theory, we just need to replace the value of

$$\mathbf{w} \Rightarrow \mathbf{V}, \qquad \mathbf{u} \Rightarrow \mathbf{U} \qquad \Rightarrow \frac{\mathbf{V}}{c} = \frac{\mathbf{U}}{b}. \tag{V-1}$$

Therefore,

$$\gamma^{-1} = \sqrt{1 - \left(\frac{\mathbf{V}}{c}\right)^2} = \sqrt{1 - \left(\frac{\mathbf{U}}{b}\right)^2} \tag{V-2}$$

$$b = \sqrt{U^2 + c^2} \Rightarrow \gamma^{-1} = \sqrt{1 - \left(\frac{\mathbf{U}^2}{U^2 + c^2}\right)} \tag{V-3}$$

$$\gamma^{-1} = \sqrt{\left(\frac{c^2}{U^2 + c^2}\right)} \Rightarrow \gamma = \sqrt{\frac{U^2 + c^2}{c^2}} = \frac{b}{c} \tag{V-4}$$

Rohrlich-Ott-Gamba-like proposal within the Einstein's dual theory

In this case, we have

$$\omega^{\mu} = \left(\gamma, \gamma\frac{\mathbf{V}}{c}\right) \Rightarrow \omega^{\mu} = \frac{b}{c}\left(1, \frac{\mathbf{U}}{b}\right) = \left(\gamma, \frac{\mathbf{U}}{c}\right) = \left(\frac{b}{c}, \frac{\mathbf{U}}{c}\right) \tag{V-5}$$

and

$$u^{\mu} = \left(\gamma, \gamma\frac{\mathbf{U}}{b}\right) = \left(\frac{b}{c}, \frac{\mathbf{U}}{c}\right) \tag{V-6}$$

Therefore,

$$u^{\mu}\omega_{\mu} = \frac{b^2}{c^2} - \frac{\mathbf{U}^2}{c^2} = \frac{b^2 - \mathbf{U}^2}{c^2} = \frac{c^2}{c^2} = 1. \tag{V-7}$$

Rohrlich-Ott-Gamba-Einstein Dual

The considered volume is

$$V_{K_{rest}} = \frac{V_{rest}}{u^\mu \omega_\mu} = V_{rest}, \tag{V-8}$$

and the corresponding $4-$vector volume is represented by

$$V^\mu_{K_{rest}} = \frac{\omega^\mu V_{rest}}{u^\nu \omega_\nu}$$

$$V^\mu_{K_{rest}} = \omega^\mu V_{rest} = u^\mu V_{rest} = \left(\gamma, \gamma \frac{\mathbf{U}}{b}\right) V_{rest} = \left(\frac{b}{c}, \frac{\mathbf{U}}{c}\right) V_{rest}. \tag{V-9}$$

PE-like proposal within the Einstein's dual theory

Let us now consider a volume generated by a Lorentz contraction in K of a body whose volume V_{rest} is at rest in K_{rest} . This volume is equal to

$$V_K = \gamma^{-1} V_{rest}. \tag{V-10}$$

in K. As we noticed before, all the points of this volume are simultaneous in K and, consequently, we can consider it as a volume at rest in K. In the Nakamura formalism, the different $4-$vectors will be described in K by

$$\omega^\mu = (1, \mathbf{0}) \qquad and \qquad u^\mu = \left(\gamma, \gamma \frac{\mathbf{U}}{b}\right) = \left(\frac{b}{c}, \frac{\mathbf{U}}{c}\right). \tag{V-11}$$

Therefore,

$$u^\mu \omega_\mu = \gamma = \frac{b}{c}. \tag{V-12}$$

PE-Einstein

The considered volume is

$$V_{K_{rest}} = \frac{V_{rest}}{u^\mu \omega_\mu} = \frac{V_{rest}}{\gamma} = \gamma^{-1} V_{rest} = \frac{c}{b} V_{rest}, \tag{V-13}$$

and the corresponding $4-$vector volume is represented by

$$V^\mu_{K_{rest}} = \frac{\omega^\mu V_{rest}}{u^\mu \omega_\mu} = \omega^\mu \frac{V_{rest}}{\gamma} = (1,0)\,\frac{V_{rest}}{\gamma} = \left(\frac{c}{b}, 0\right) V_{rest}. \tag{V-14}$$

Therefore, in this case, the differential of the volume is:

$$dV^\mu_{K_{rest}} = \frac{\omega^\mu dV_{rest}}{u^\mu \omega_\mu} = \omega^\mu \frac{dV_{rest}}{\gamma} = (1,0)\,\frac{dV_{rest}}{\gamma} = \left(\frac{c}{b}, 0\right) dV_{rest}. \tag{V-15}$$

Landsberg Proposal

If we consider that we are thinking in define the instantaneity in the rest frame of the system where $d\tau = dt$ and the volume $V = V_0$, we have

$$\omega^\mu = (1, \mathbf{0}) \qquad and \qquad u^\mu = (1, 0)\,. \tag{V-16}$$

Therefore,

$$u^\mu \omega_\mu = 1. \tag{V-17}$$

The considered volume is

$$V_{K_{rest}} = \frac{V_{rest}}{1} = V_{rest}, \tag{V-18}$$

and the corresponding $4-$vector volume is represented by

$$V^\mu_{K_{rest}} = (1, \mathbf{0})\, V_{rest} = (1, \mathbf{0})\, \frac{V_{rest}}{\gamma} = (1, 0)\, V_{rest}. \tag{V-19}$$

Therefore, in this case, the differential of the volume is:

$$dV^\mu_{K_{rest}} = (1, \mathbf{0})\, dV_{rest}. \tag{V-20}$$

Concluding Remarks

We can define all the instantaneity in K_0 and the time is the same τ for all the system and the difference will consist in where is the the volume at rest.

Concluding Remarks

The Proper Time in Redefined Relativistic Thermodynamics has been defined and consequently for each system of particles if we can calculate or describe the motion of the canonical center of mass described by Ec. (IV-7), we can describe in a physical form all the thermodynamics of a system in equilibrium. Many applications can be done with this result since as we notice in the introduction the reference frame such that the black-body which carries on the $2.7K$ radiation background is at rest, corresponds to the Hydra-Centaur direction with a speed of $627km/sec$. Therefore, the three proposals can be used to deal with a system of particles.

End

However, PE proposal seems to be the most adequate technique to deal with a thermodynamic system since all the quantities are defined as invariant but the big problem consists of defining the proper time reference frame. THANK YOU FOR YOUR TIME

References

- Planck M 1908 Ann. Phys. (Leipzig) 26 1
- Einstein A 1907 Jahrb. F. Rad. U Elekt. 4 411
- Tolman R C 1934 Relativity Thermodynamics and Cosmology (Oxford: Oxford University Press)
- Pauli W 1958 Theory of Relativity (London: Pergamon)
- Liu C 1982 Br. J. His. Sci. 25 185
- Ott H 1963 Z. Phys. 175 70
- Balescu R Physica 1968 40 309
- Arzeliès H 1968 Thermodynamique Relativiste et Quantique (Gauthier Villars, Paris)
- Rorhlich F 1966 Nuovo Cimento B 45 6200
- Landsberg P T 1966 Proc. Phys. Soc. 89 1007

- Landsberg P T 1980 Phys. Rev. Lett. 45 149
- Sieniytycz S 1998 Phys. Rev. E 58 7027
- Landsberg P T and Matsas G E A 1996 Phys. Lett. A 223 401
- Landsberg P T and Matsas G E 2002 A Physica A 340 92
- Unruh W C 1976 Phys. Rev. D 14 870 DeWitt B 1979 General Relativity (Cambridge: Cambridge University Press)
- Ares de Parga G, López-Carrera B and Angulo-Brown F 2005 J. Phys. A: Math. Gen. 38 2821
- Landau L D and Lifshitz E M 1958 Statistical Physics (London: Pergamon) Sec. 15
- T D Nakamura, Phys. Lett. A 352 (2006) 175; T D Nakamura, Space Sci. Rev. 122 (2006) 271 278.
- N G van Kampen, Phys. Rev. 173 (1968) 295.

- R.N. Bracewell, E.K. Conklin, Nature 219 (1968) 1343; P.J.E. Peebles, D.T. Wilkinson, Phys Rev. 174 (1968) 2168
- G.R. Henry, R.B. Feduniak, J.E. Silver, M.A. Peterson, Phys. Rev. 176 (1968) 1451.
- B.S. DeWitt, in: S.W. Hawking, W. Israel (Eds.), General Relativity, Cambridge University Press, Cambridge, 1979.
- Ares de Parga, G.; López-Carrera, B. Redefined relativistic thermodynamics based on the Nakamura formalism. Physica A 2009, 388, 4345–4356.
- Lineweaver C H 2000 In Microwave Background Anisotropies (Editions Frontiers, Git-sur-Yvette, France, astro-ph/9609034) Liddle A R and Lyth D H 2000 Cosmological In°ation and large-scale Structure (Cambridge: Cambridge University Press) p. 246

- Moshinsky M 1987 Einstein en la Ciencia del Futuro, Albert Einstein: Perfiles y Perspectivas (Editorial Nueva Imagen, México-Caracas-Buenos Aires, UNAM) pp 387-391
- Curie D G Jordan T F Sudarshan E C G 1963 Rev. Modern Phys. 35 (2) 350
- Balescu R, Kotera T 1967 Physica 33 558
- H. Goldstein, Classical Mechanics, 3rd ed., Addison Wesley, New York, 2000, p. 408. 25
- S.S. Schweber, An Introduction to Relativistic Quantum Field Theory, Harper and Row, New York, 1962, p. 21
- López-Carrera B Ares de Parga G 2008 Relativistic transformation of the canonical distribution function in relativistic statistical mechanics. Physica A 387 1099–1109

Ares de Parga G and López-Carrera B Relativistic Statistical Mechanics vs Relativistic Thermodynamics Entropy 2011, 13, 1664-1693; doi:10.3390/e13091664

Gill, T L, Zachary W W Lindesay J The classical electron problem. Found. Phys. 31, 1299–1354 (2001)

Ares de Parga G Gill T L Zachary W W 2013 The Thomas program and the canonical proper-time theoryJournal of Computational Methods in Sciences and Engineering 13 117–134

Gill T L, Ares de Parga G The Einstein dual theory of relativity Adv. Stud. Theor. Phys. 13(8) 337–377 (2019)

Gill T L Ares de Parga G Morris T and Wade M, Dual Relativistic Quantum Mechanics I, Foundations of Physics (online) DOI 10.1007/s10701-022-00607-4, (2022).

- Gill T L Ares de Parga G Foundations for QED, Feynman operator calculus, Dyson conjectures, and Einstein's dual theory 2023 Journal of Physics: Conference Series 2482 012015 doi:10.1088/1742-6596/2482/1/012015
- E. Fermi, Nuovo Cimento 25 (1923) 159. Reprinted in E. Fermi, Note e Memorie (collected papers) (University of Chicago Press, Chicago, 1962), Vol. I.
- J.L. Synge, Relativity, The Special Theory, North-Holland, Amsterdam, 1956, p. 429. Appendix D.
- L.D. Landau, E.M. Lifshitz, Classical Theory of Fields, Pergamon, London, 1962, Section 32.
- F. Rohrlich, Classical Particles, Foundations of their Theory, Addison-Wesley, Reading, 1965, p. 282. Appendix A1 6.
- A. Gamba, Am. J. Phys. 35 (1967) 83.

Gonzalez-Narvaez R Ares de Parga A M, Ares de Parga G 2017 Mixing of relativistic ideal gases with relative relativistic velocities *Ann. Phys.* 376, 391–411